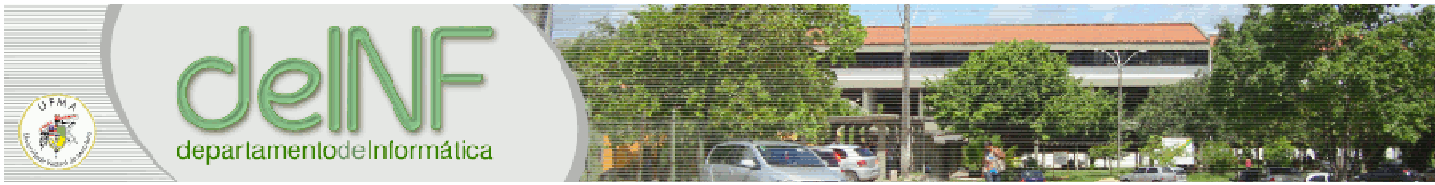
Projeto e Análise de Algoritmos

Aula 1: Algoritmos de Ordenação

Prof. Carlos de Salles

Terças-feiras, 8h20 às 11h10



Algoritmos de Ordenação

- Insertsort
- Mergesort
- Heapsort
- Quicksort

Algoritmos de Ordenação

- Dado um vetor **v** com **N** elementos, a ordenação consiste em organizar todos esses **N** elementos em uma ordem (não-crescente, não-decrescente etc)
- Exemplo:
  - v = {1, 9, 8, 5, 3, 7, 4}
    - Depois de ordenado de forma não-decrescente, v fica:
    - v = {1, 3, 4, 5, 7, 8, 9}

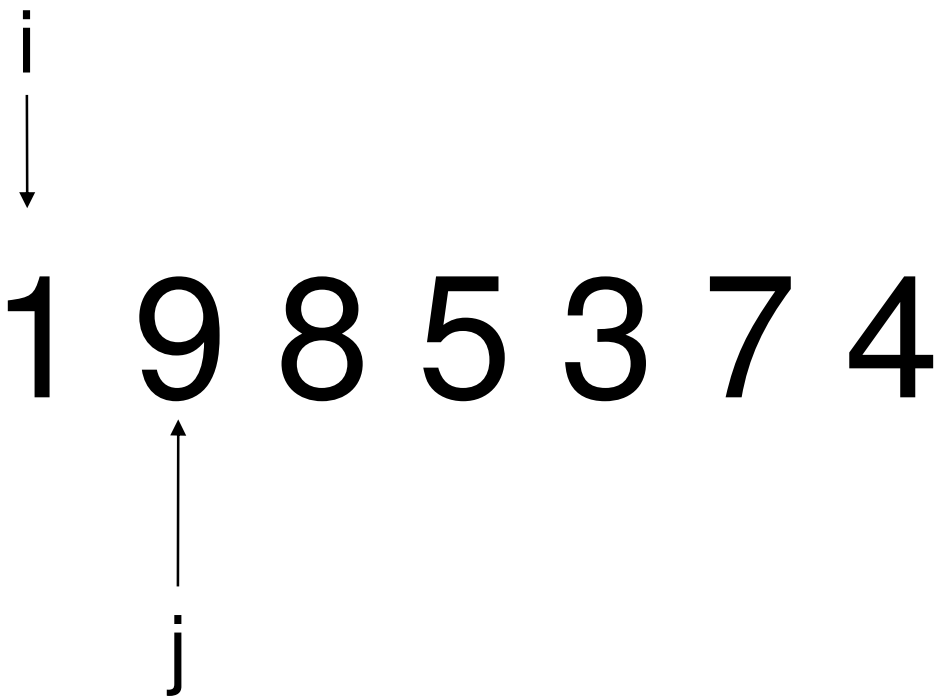
Insert Sort

- Outros nomes:
  - Bubble sort
  - Selection sort
- Consite em posicionar o menor ou o maior elemento em seu lugar correto **N** vezes consecutivas

Insert Sort

```
for i=1,N do
    for j=i+1,N do
        if v[i]>v[j] then
            v[i],v[j] = v[j],v[i]
        end
    end
end
```

Insert Sort



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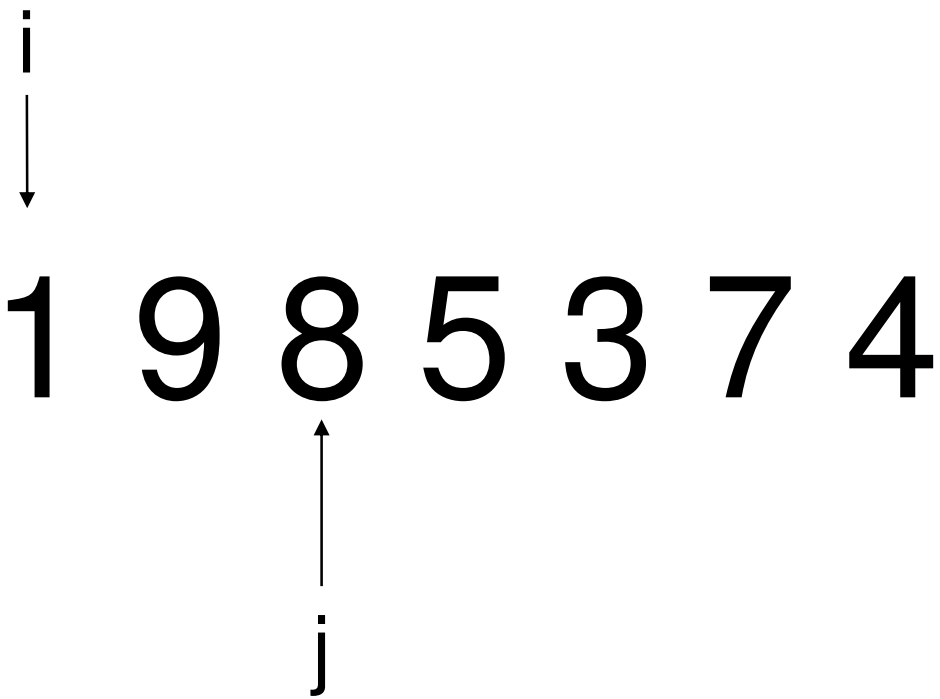

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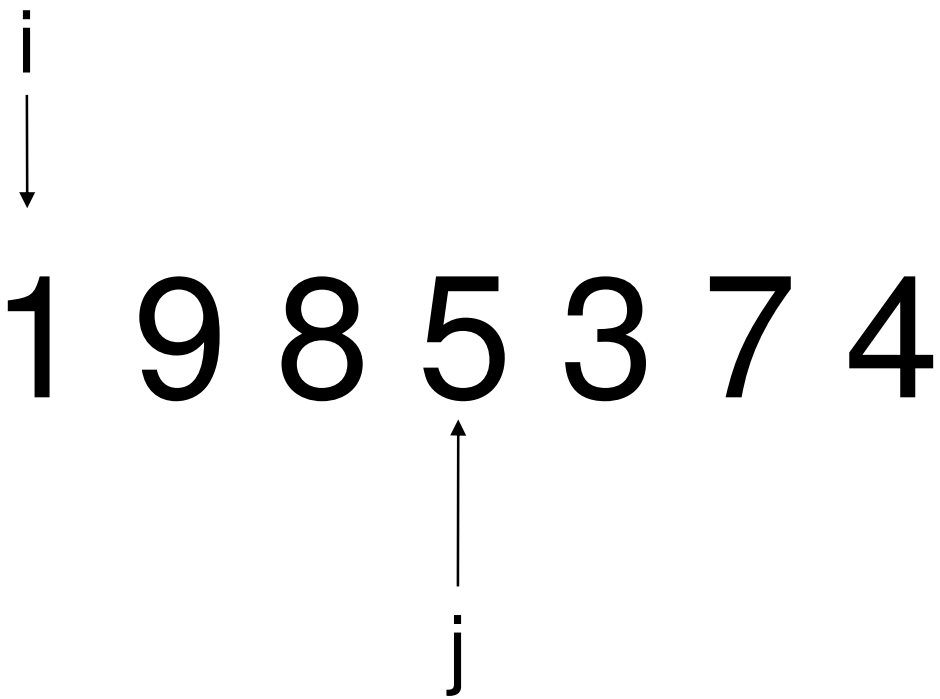

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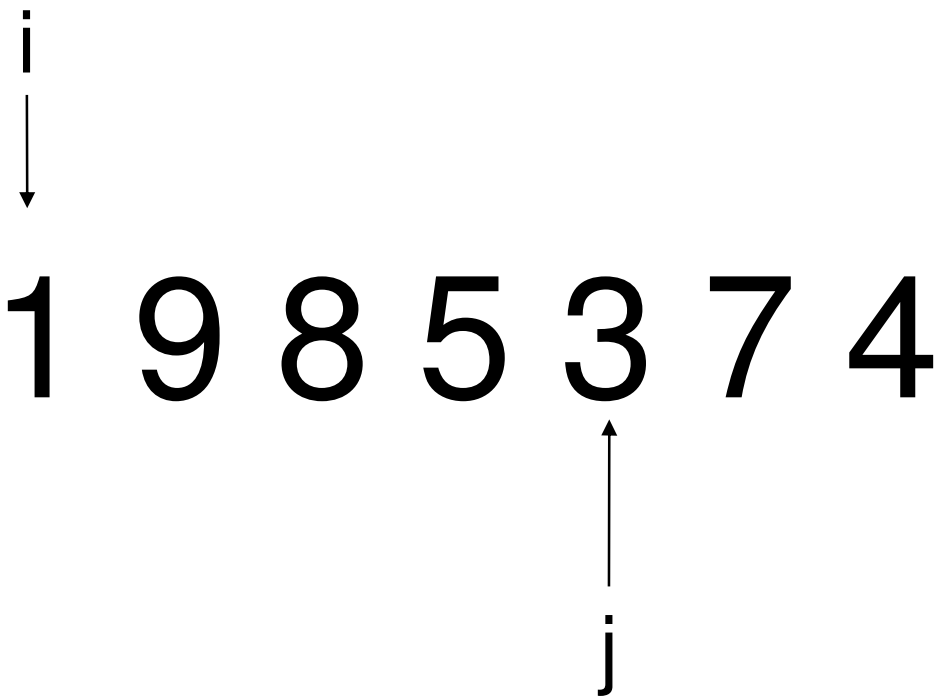

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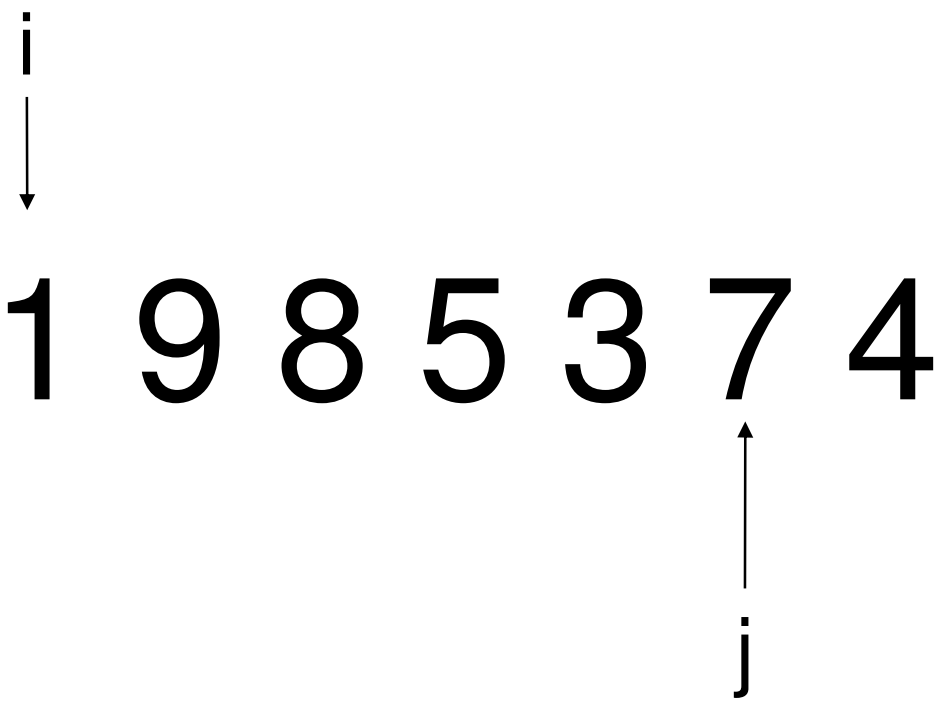

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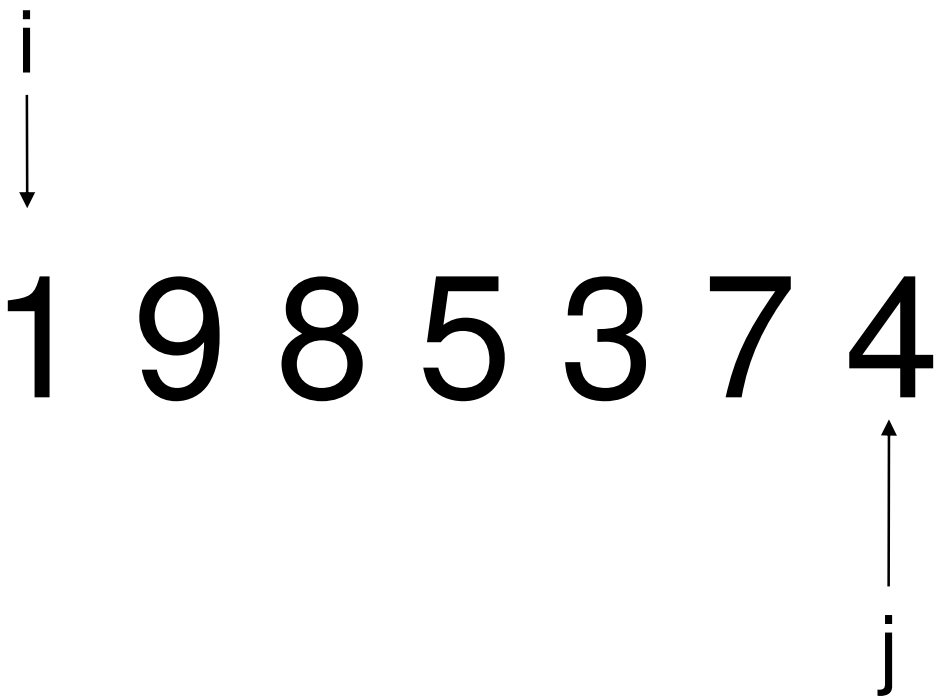

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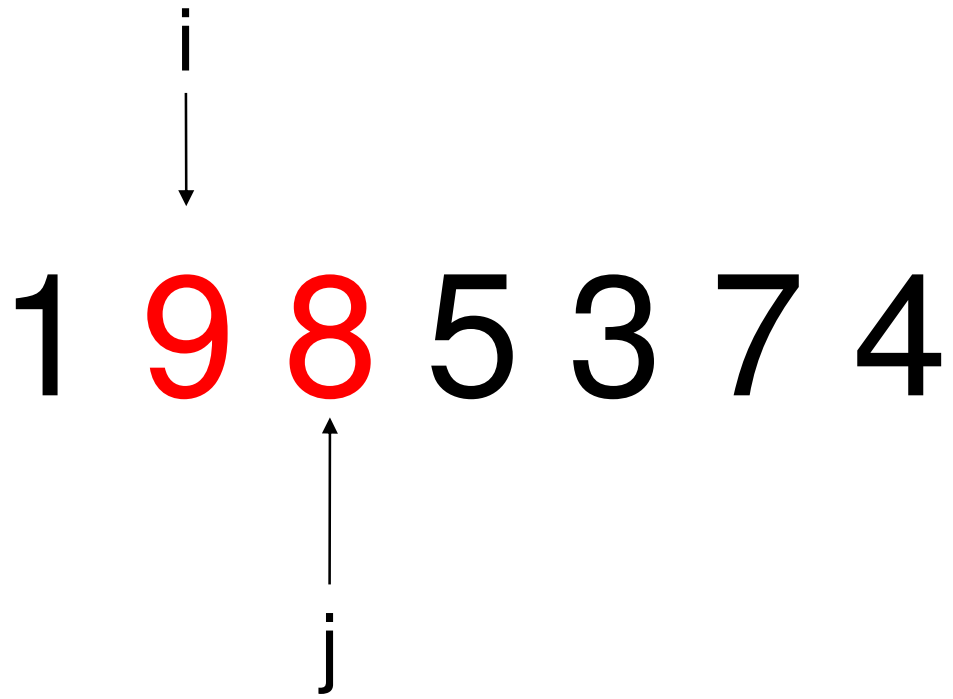

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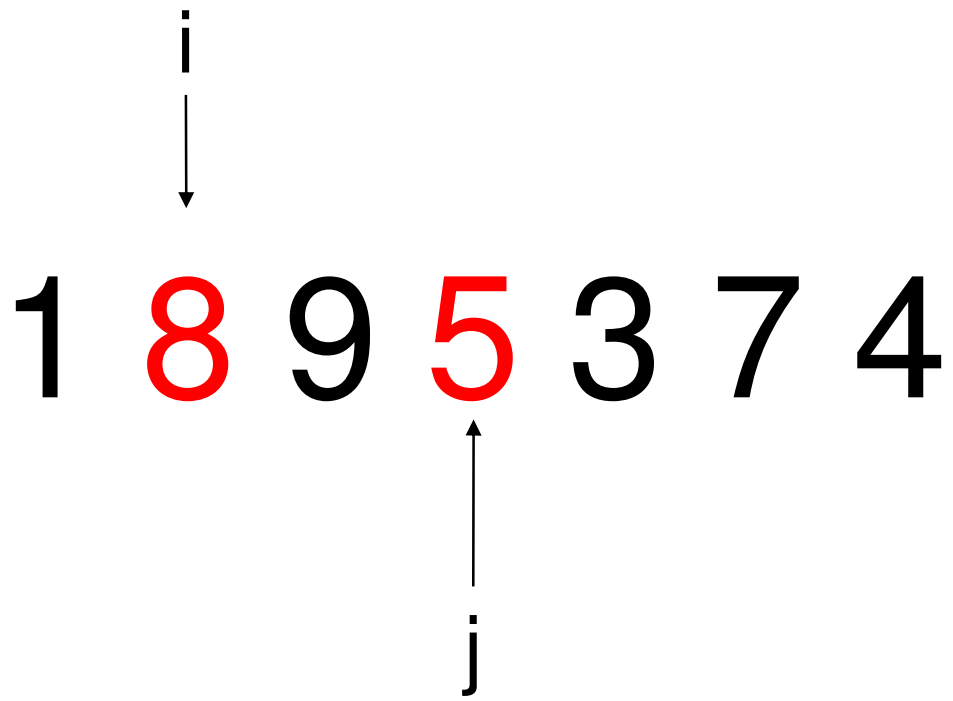

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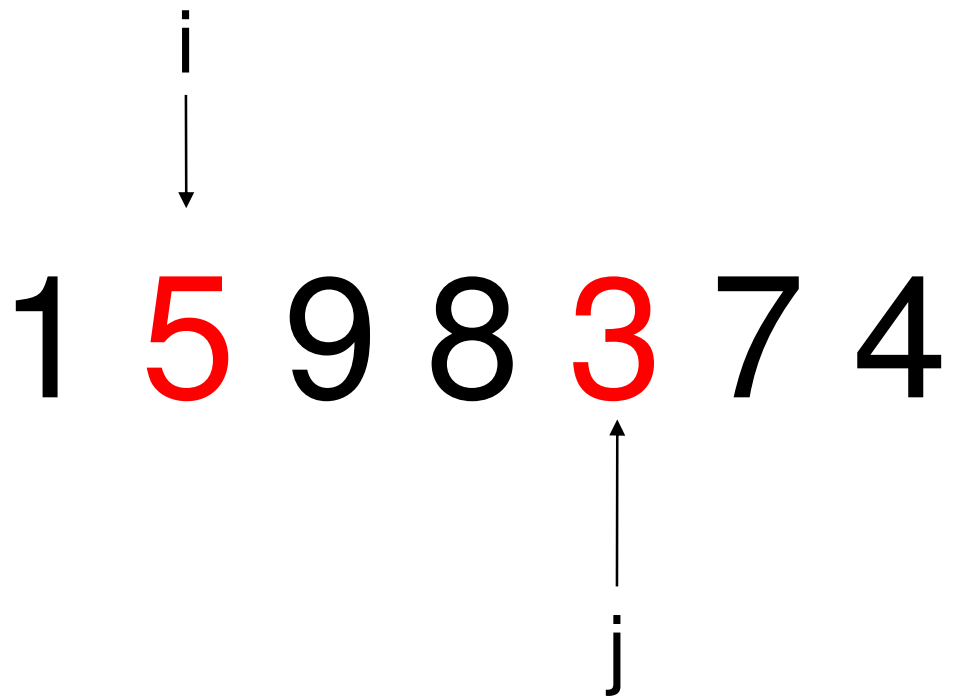

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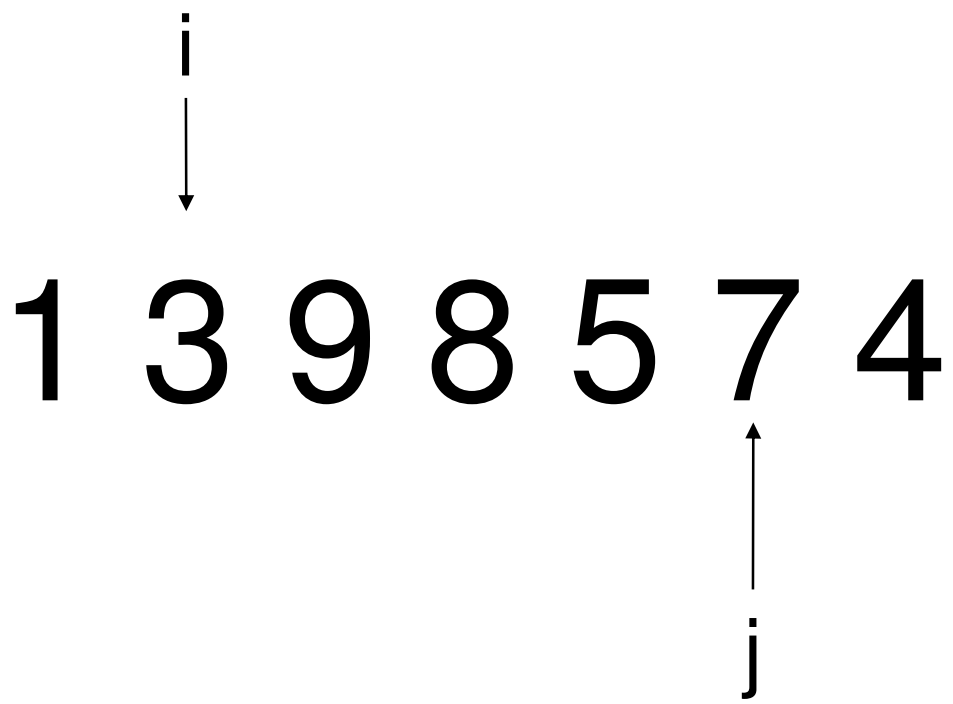

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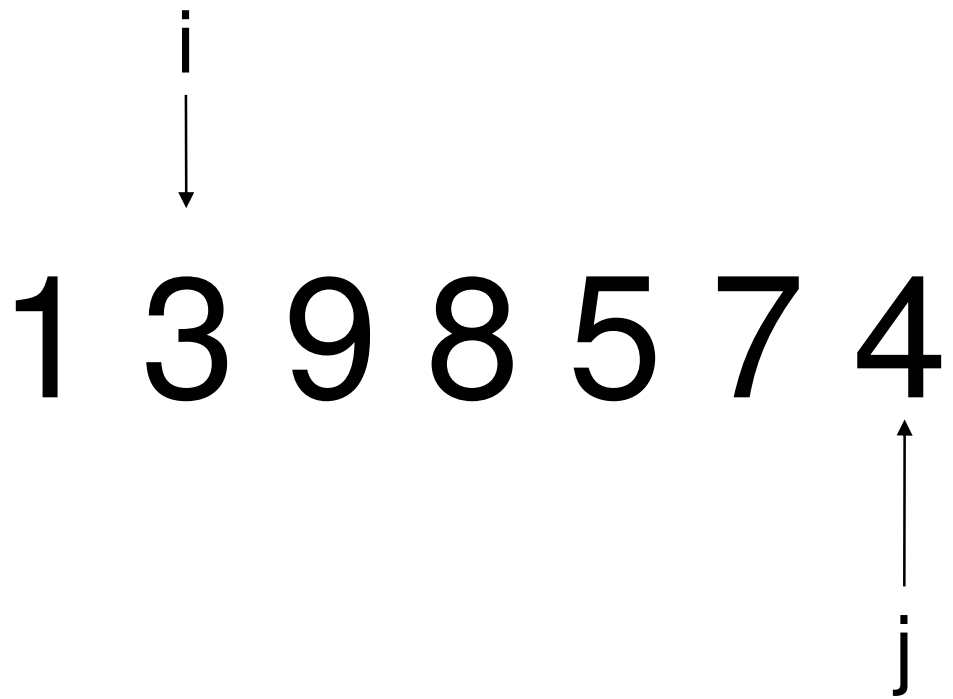

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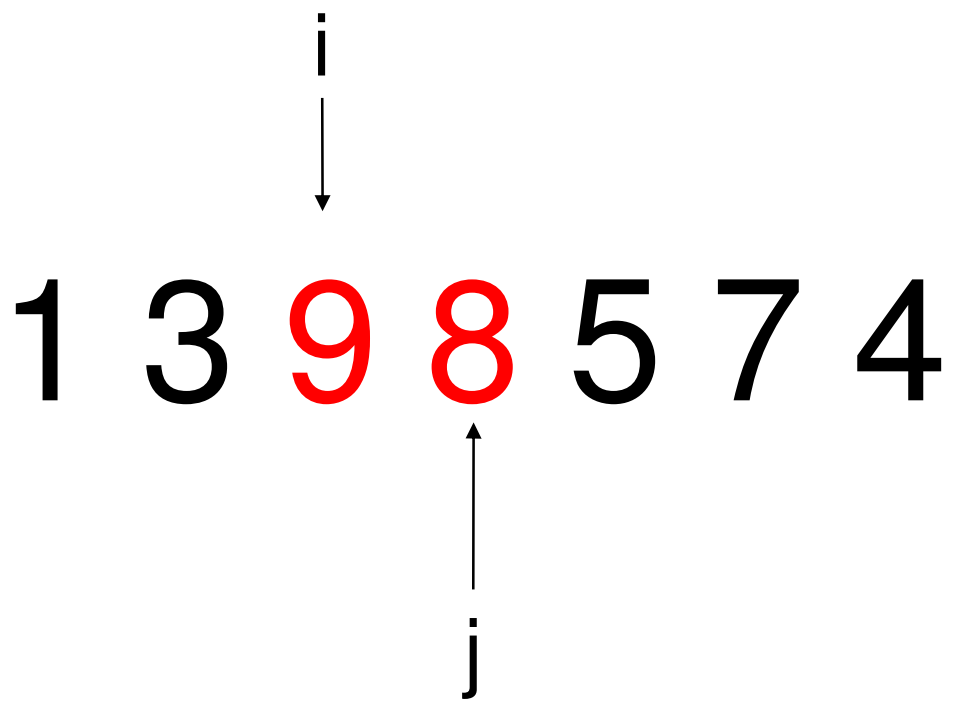

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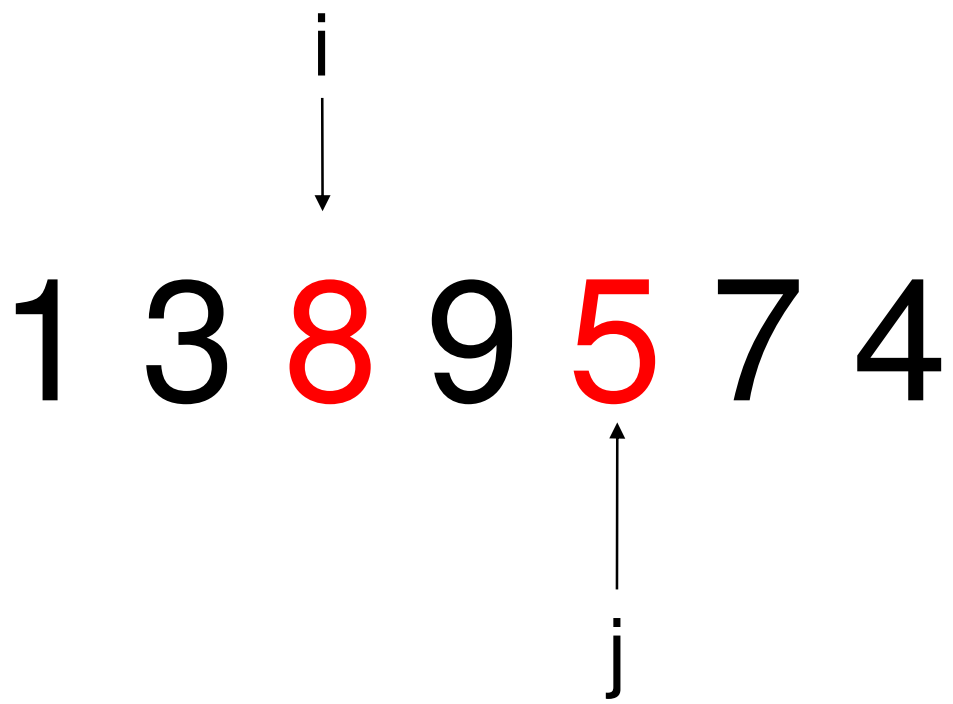

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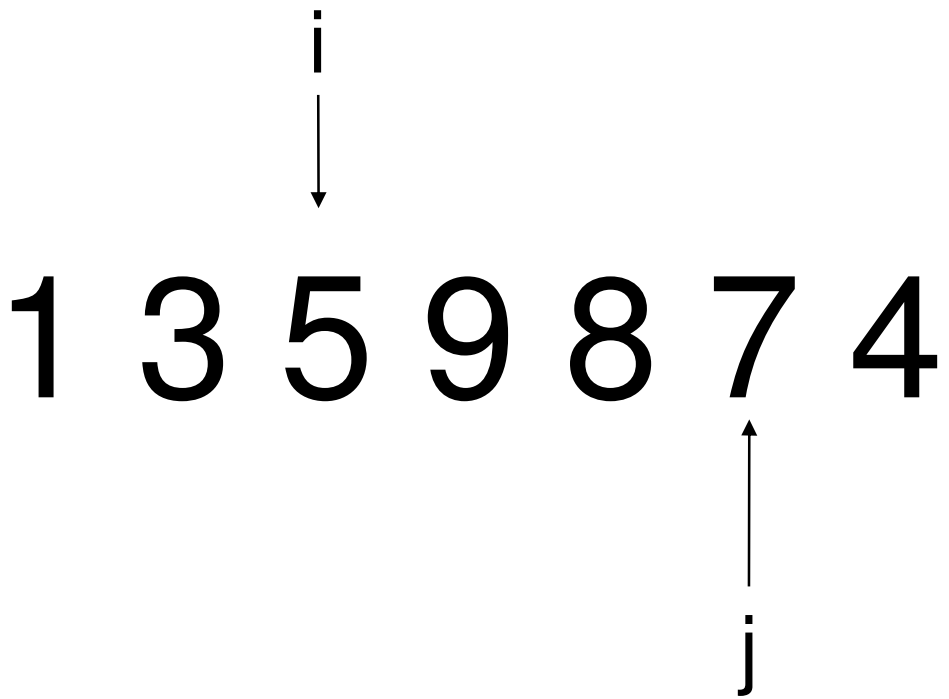

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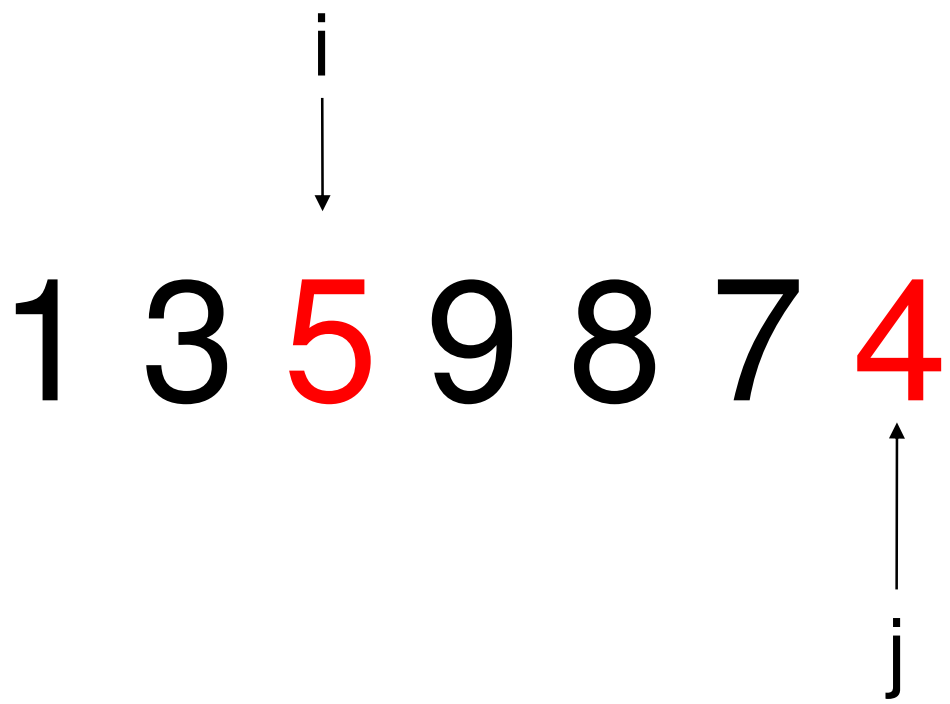
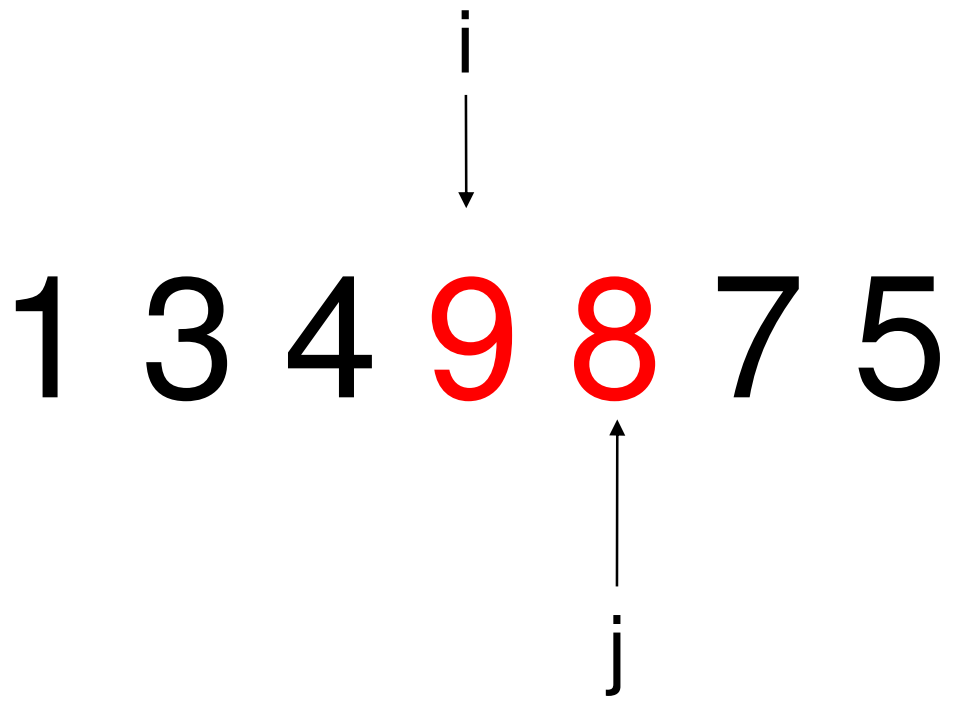


Insert Sort



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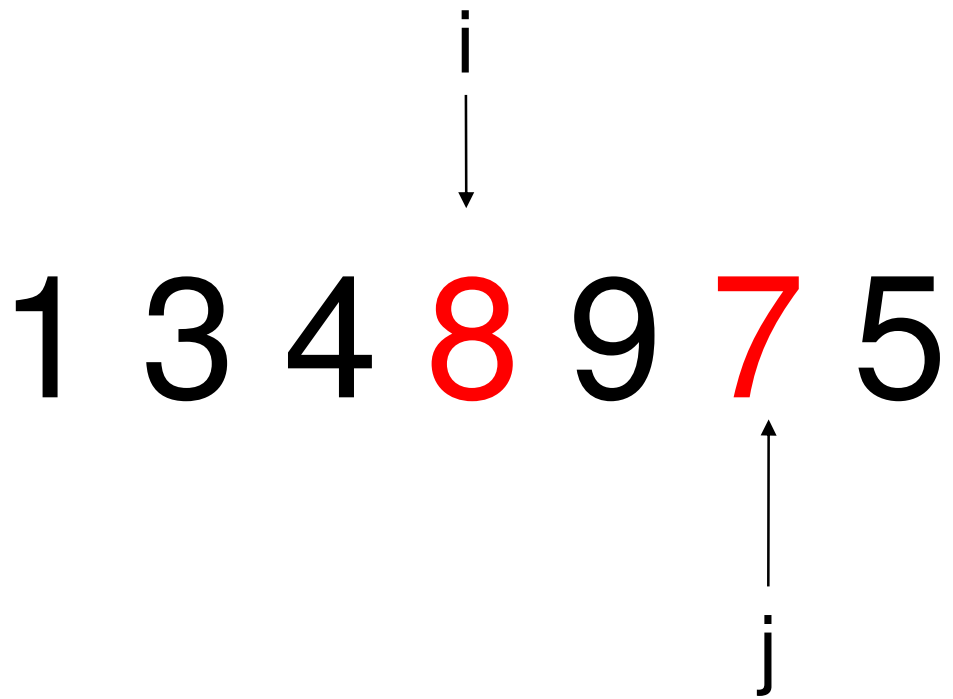

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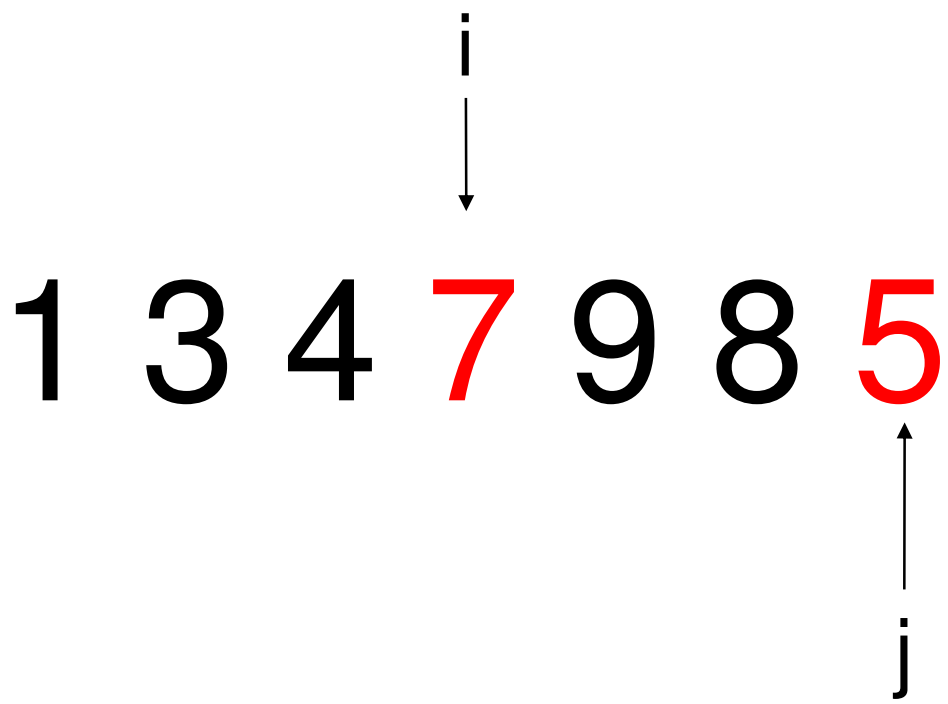

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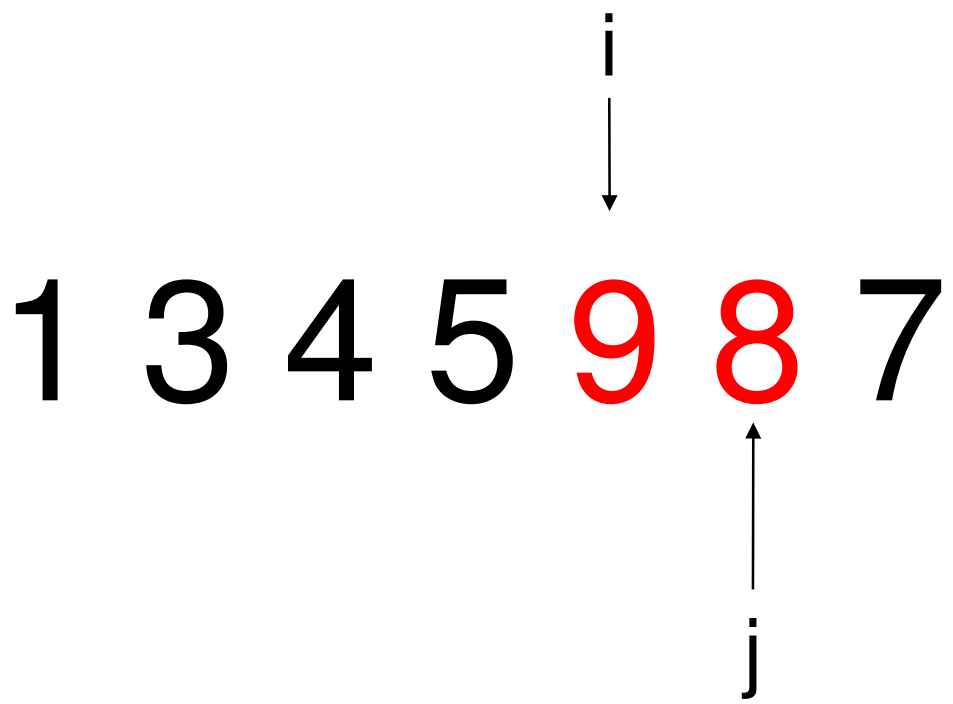

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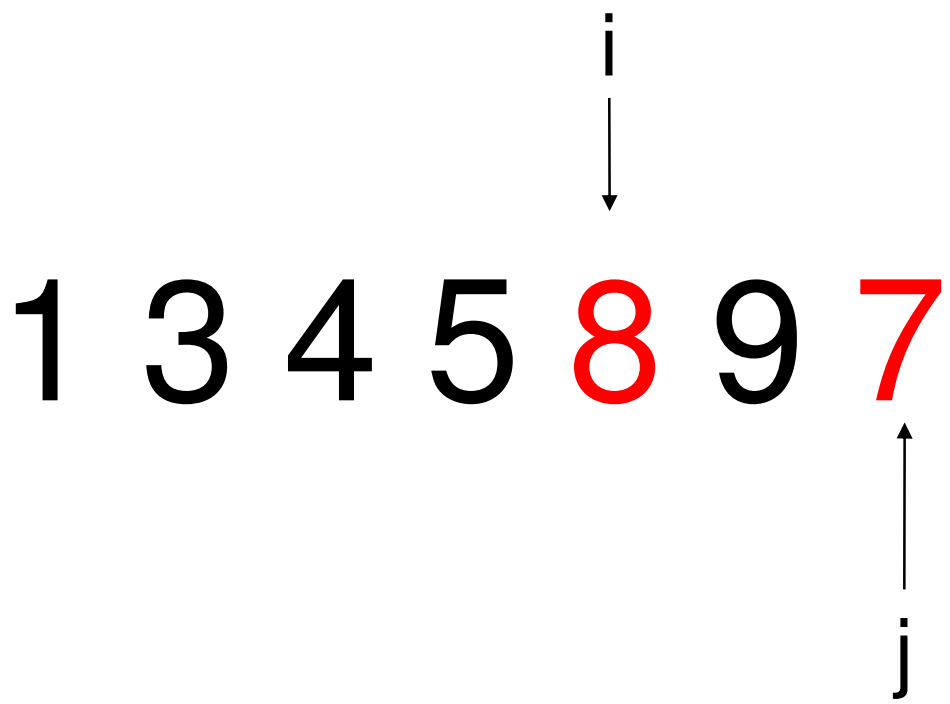

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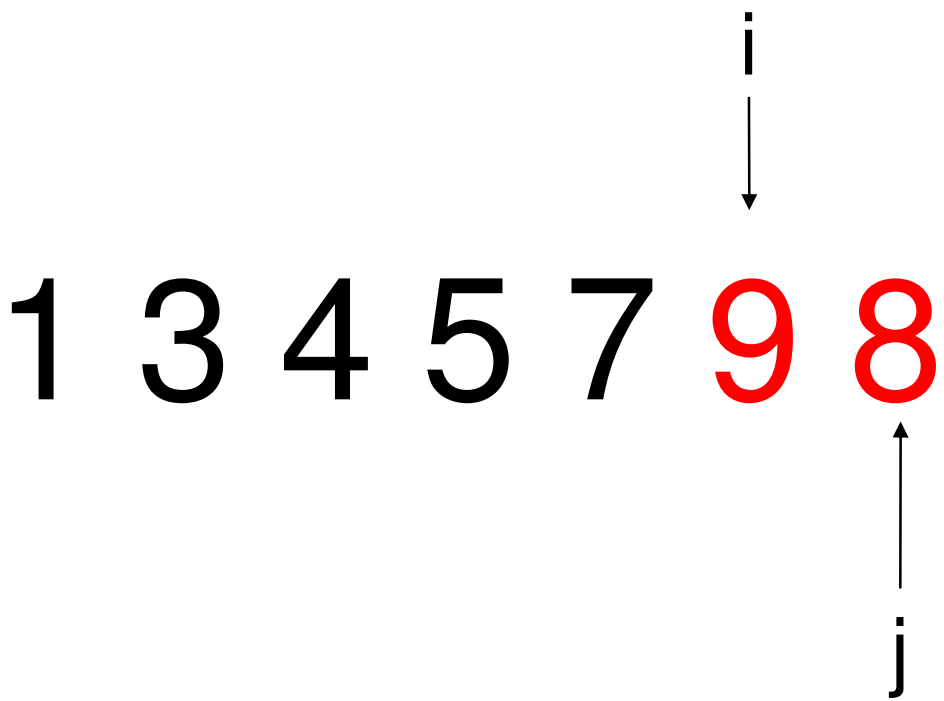


Insert Sort

1 3 4 5 7 8 9

Insert Sort

1 3 4 5 7 8 9

Número de operações:
(N-1) + (N-2) + (N-3) + … + 3 + 2 + 1

$$\frac{N*(N-1)}{2}$$

Insert Sort

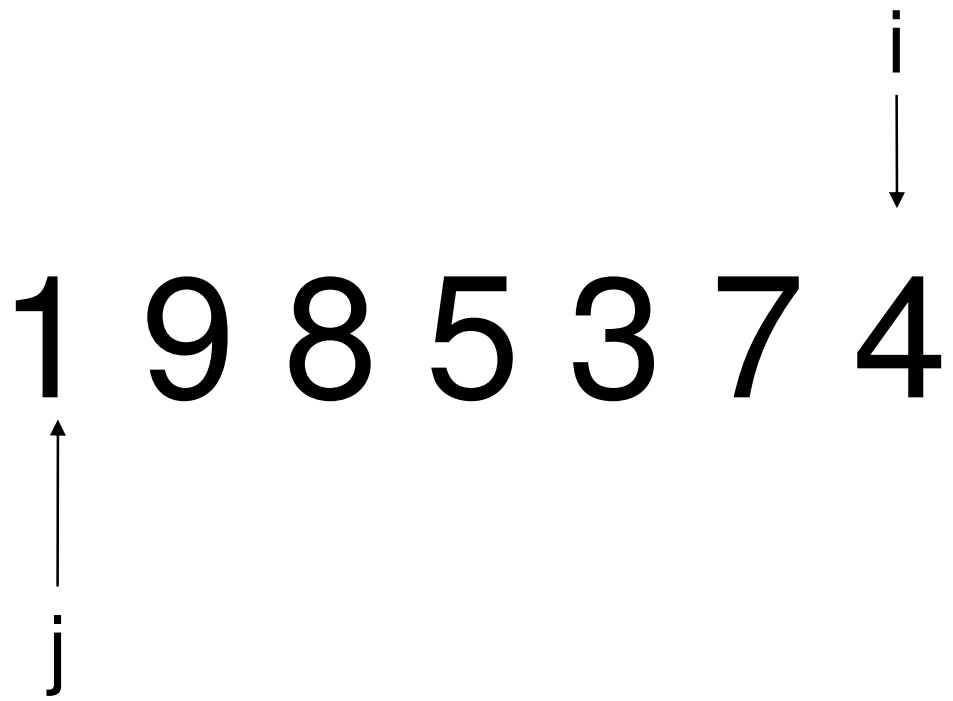
- Ideia:
  - Se o vetor já estiver ordenado em alguma interação, seria interessante detectar isso e parar o algoritmo.
- Como detecto se um vetor está ordenado?

```
ordenado = true
for i=1,N do
    if v[i]>v[i+1] then ordenado = false end
end
```

Insert Sort (melhoria)

```
for i=N,2,-1 do
    trocas = false
    for j=1,i-1 do
        if v[j]>v[j+1] then
            v[j],v[j+1] = v[j+1],v[j]
            trocas = true
        end
    end
    if not trocas then break end
end
```

Insert Sort (melhoria)



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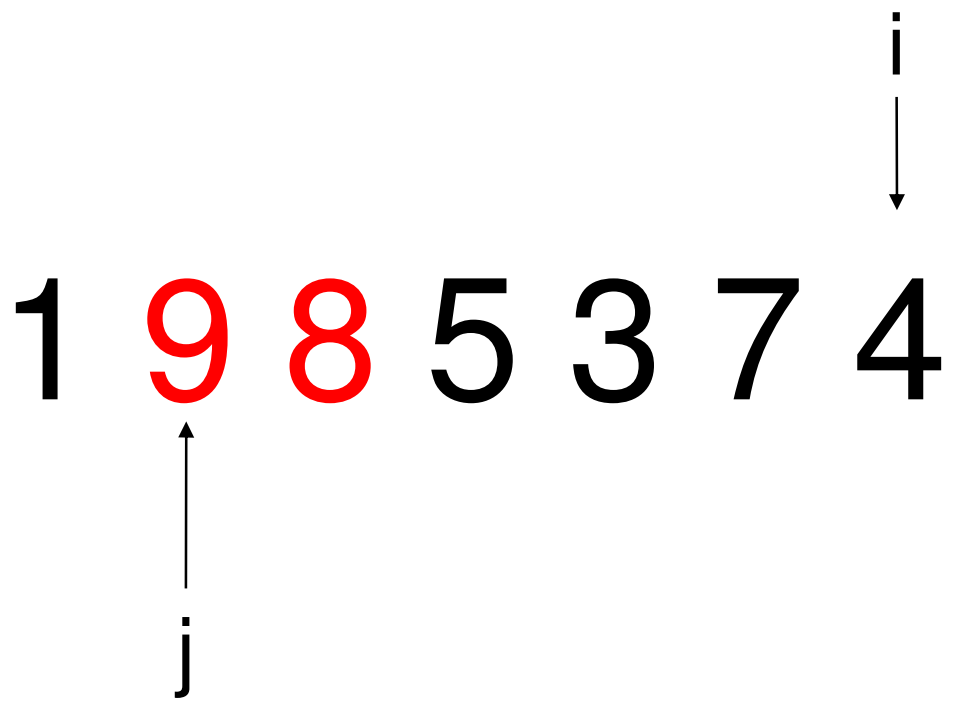

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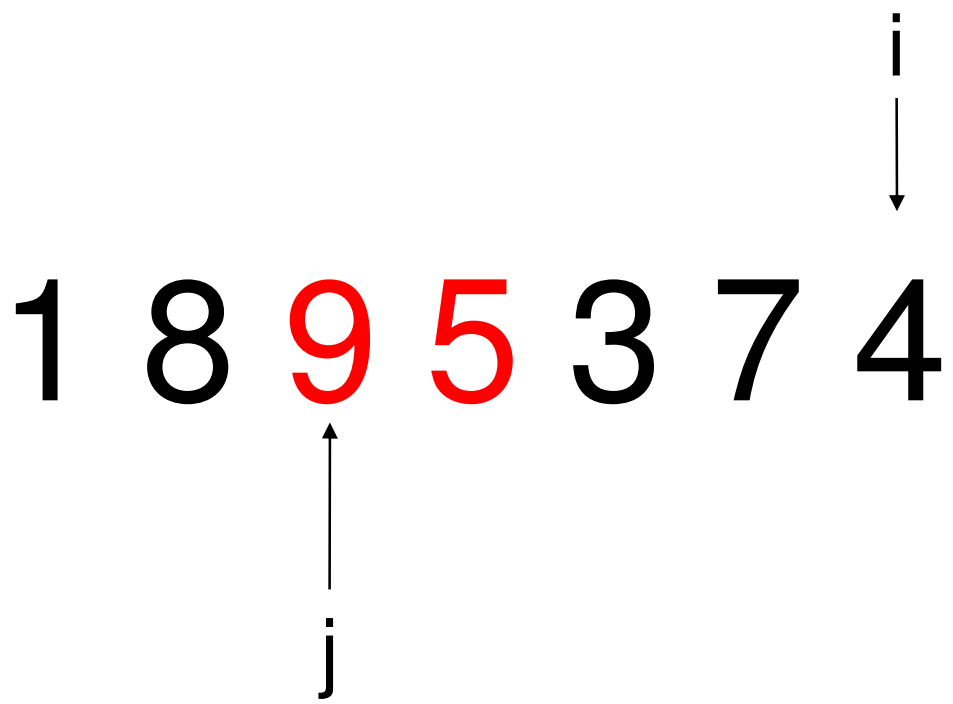

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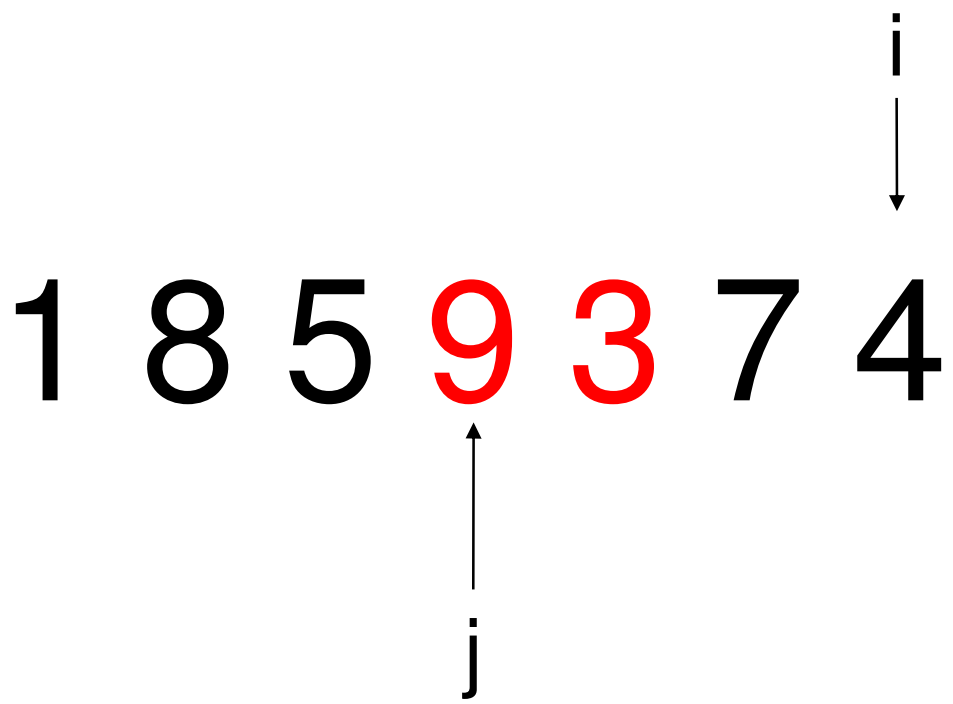

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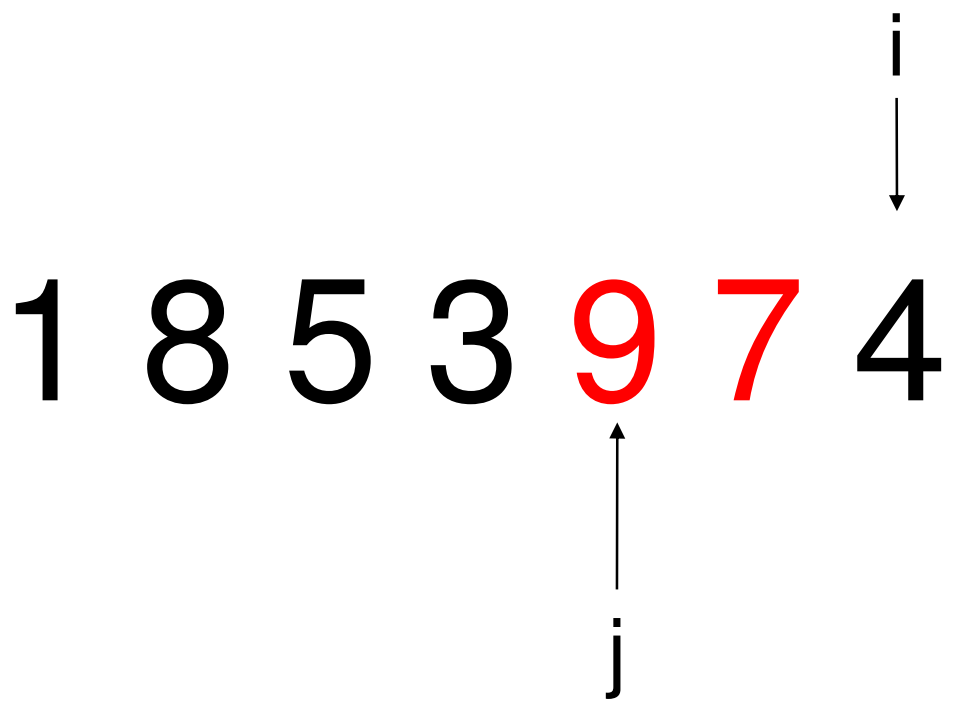

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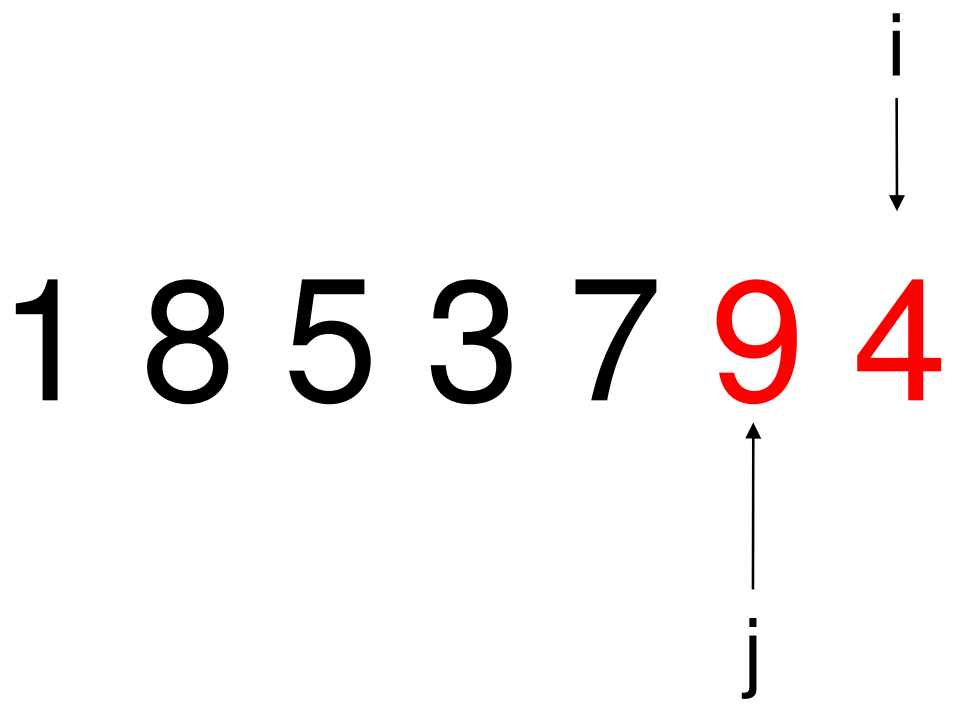

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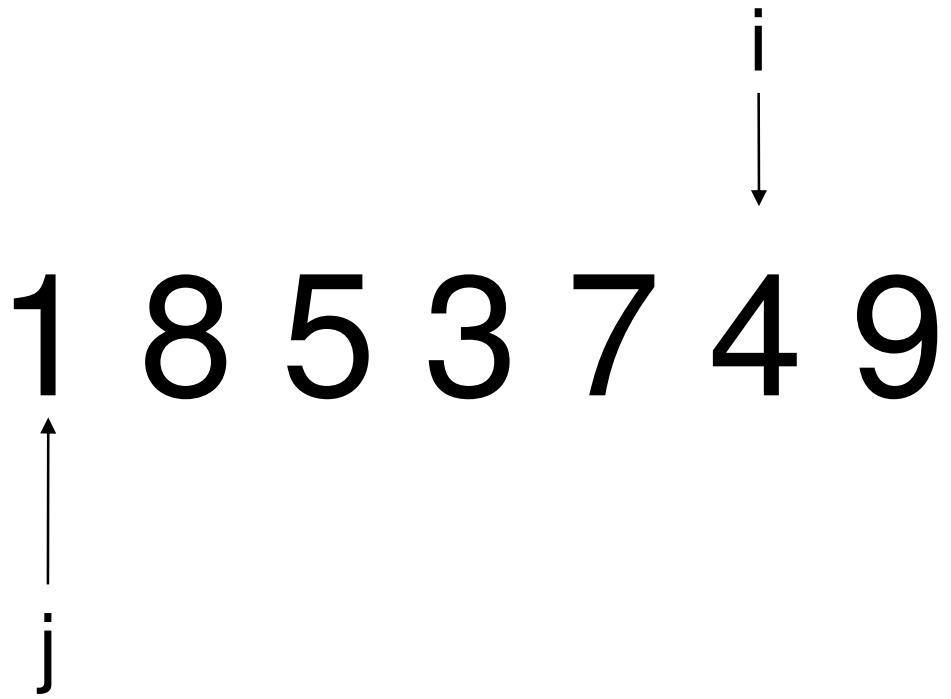

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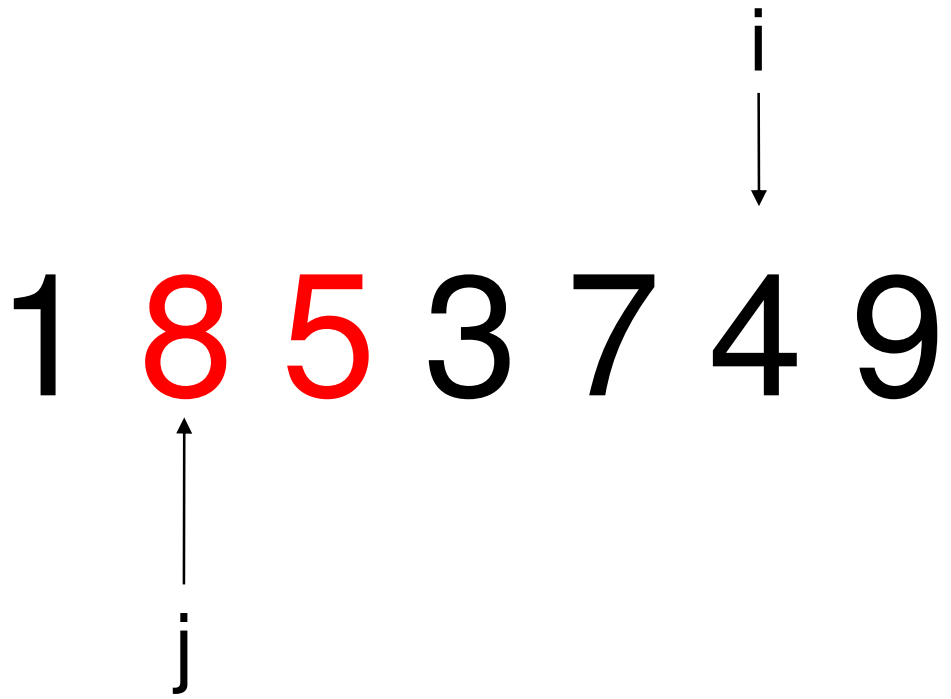

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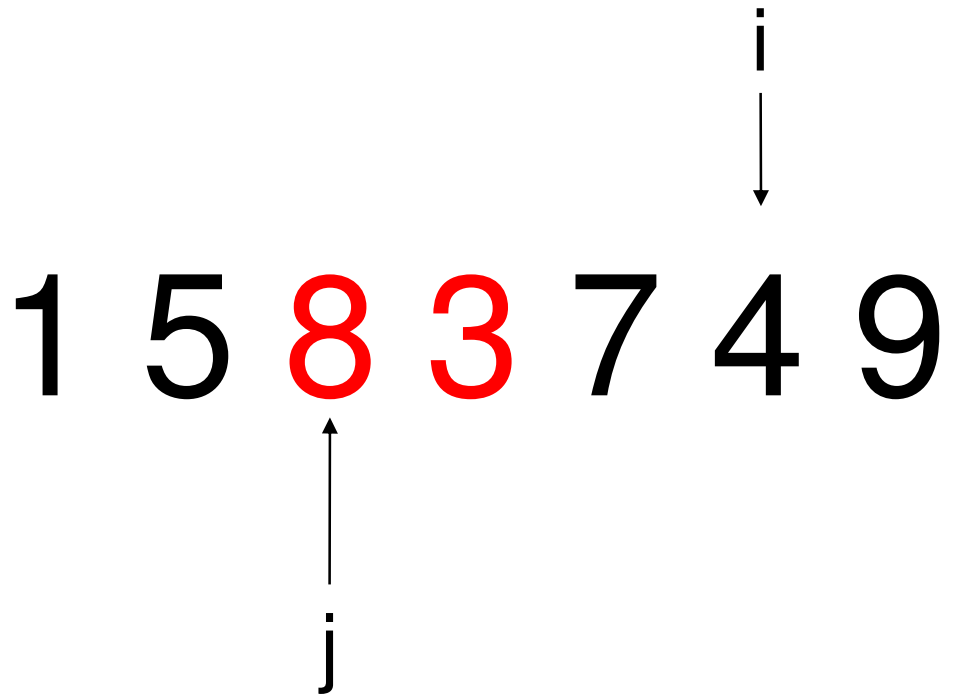

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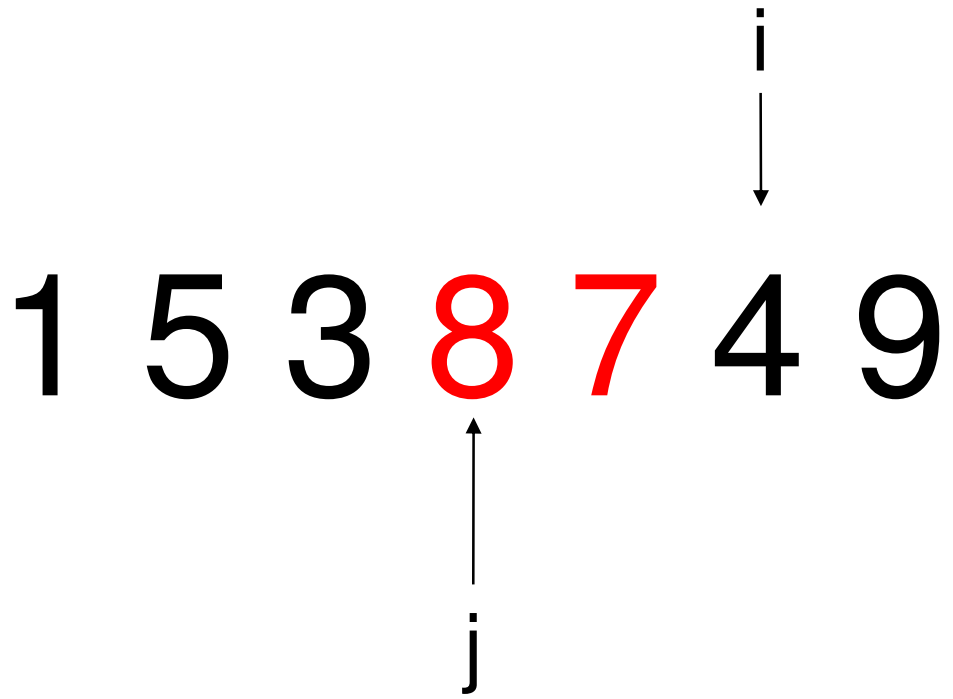

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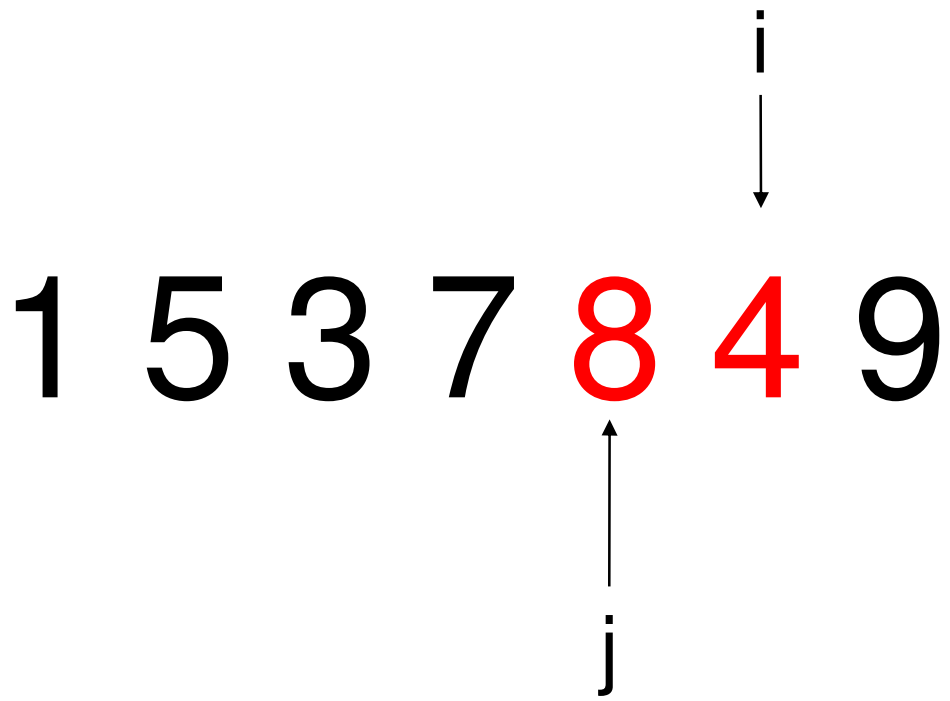

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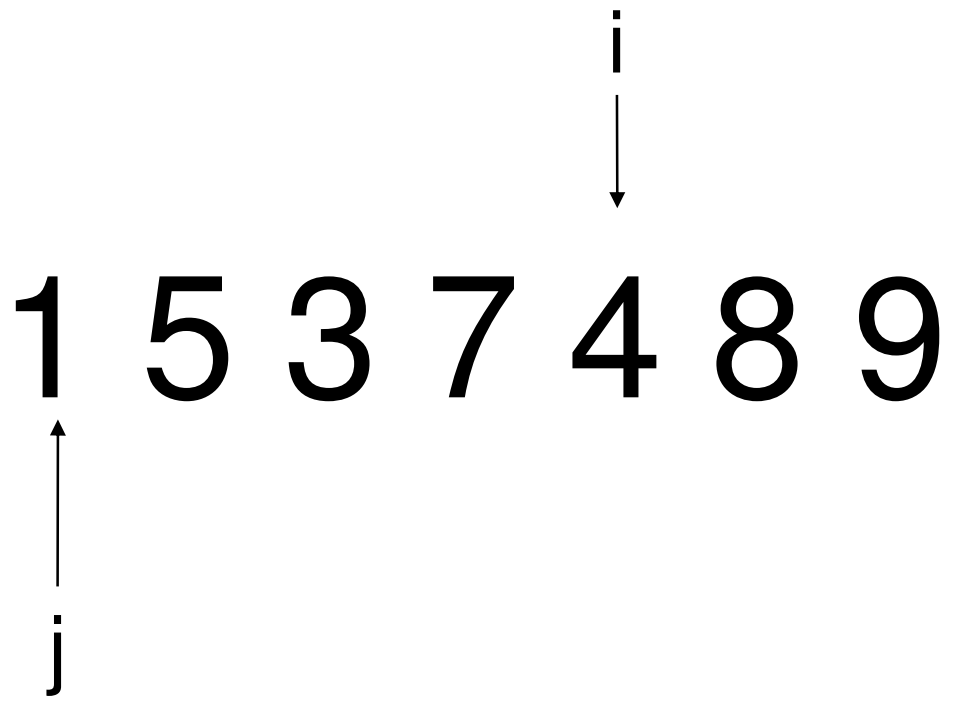

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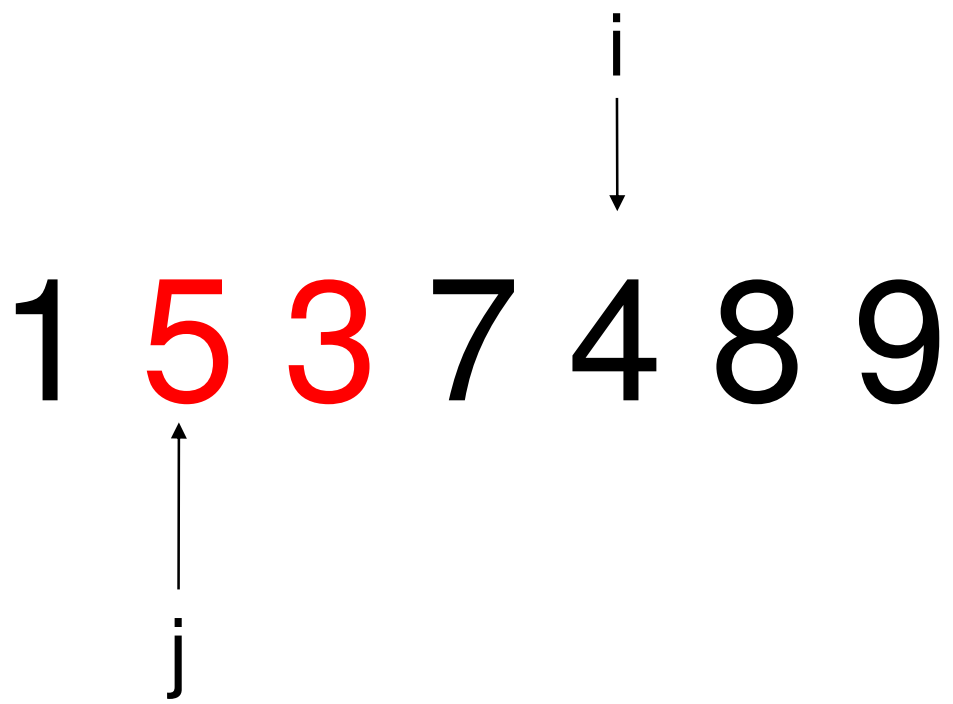

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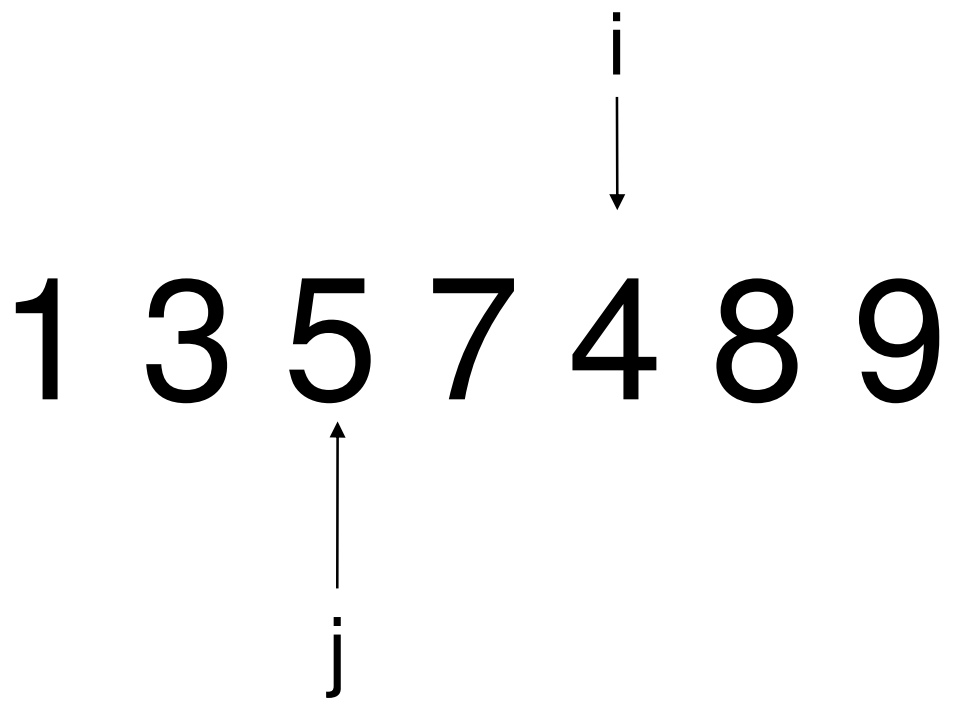

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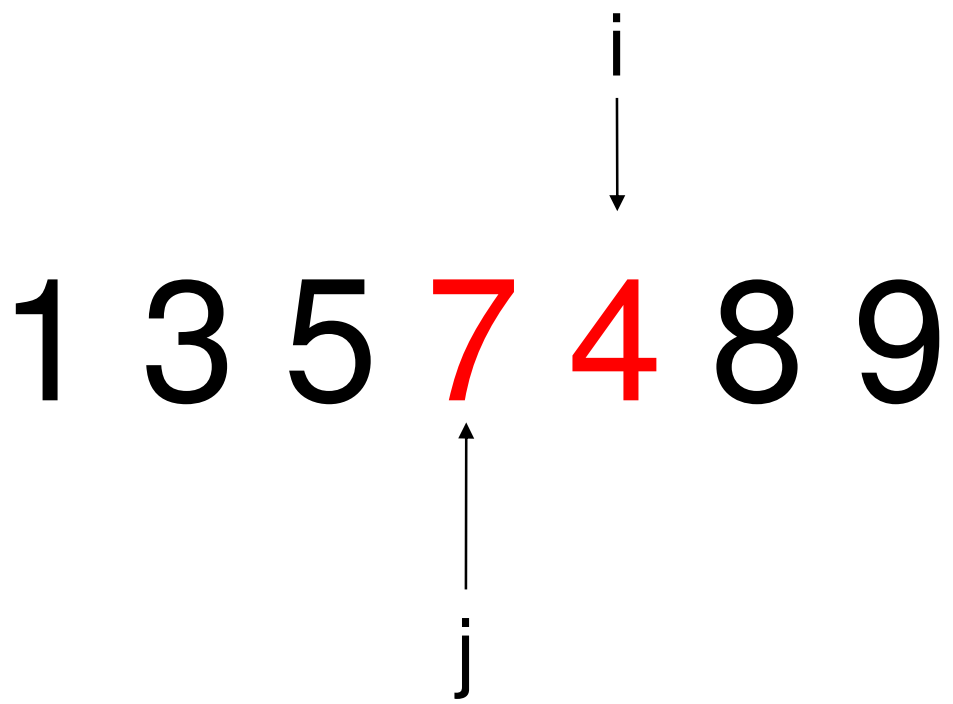

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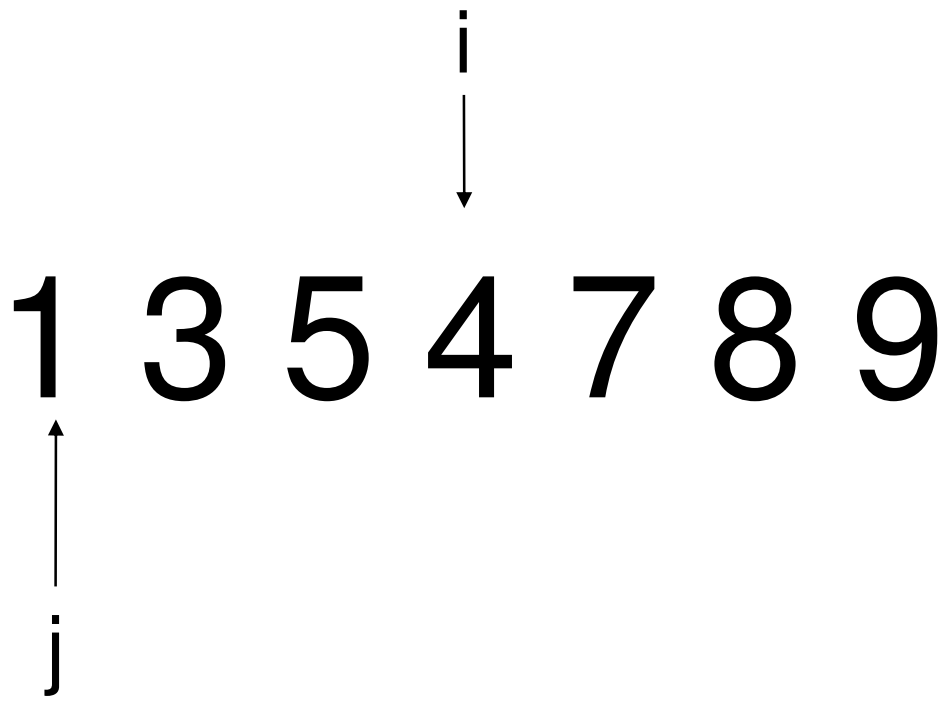

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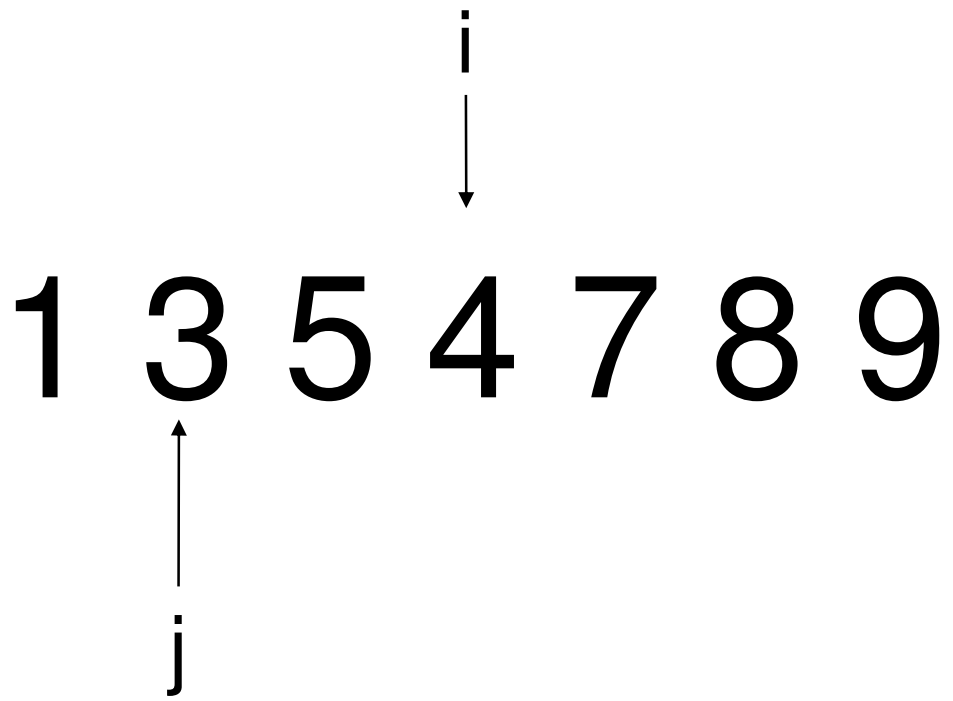

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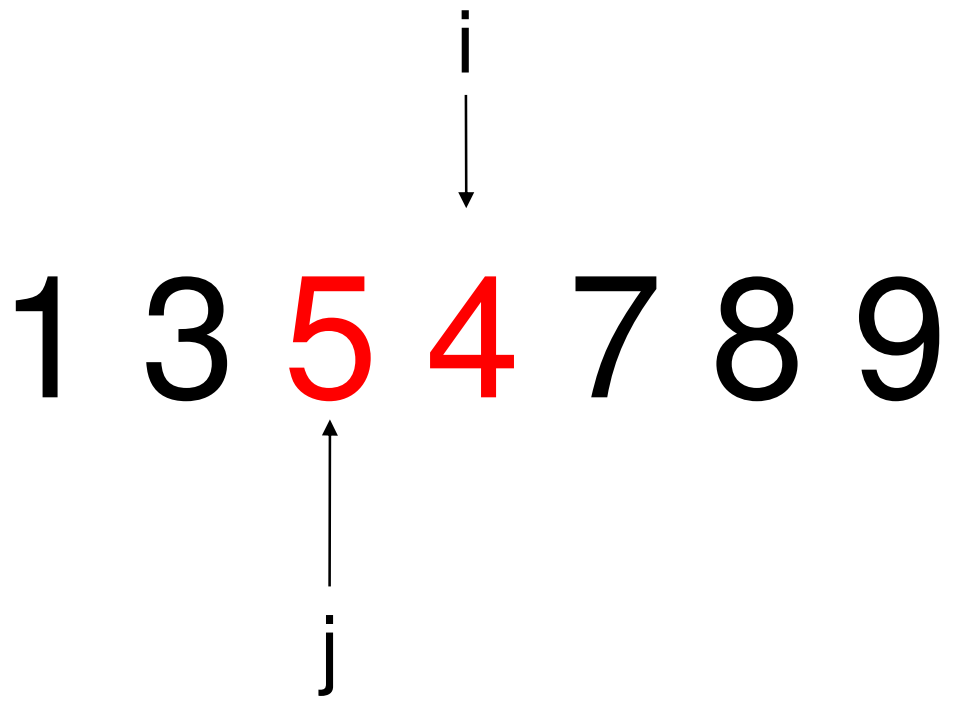

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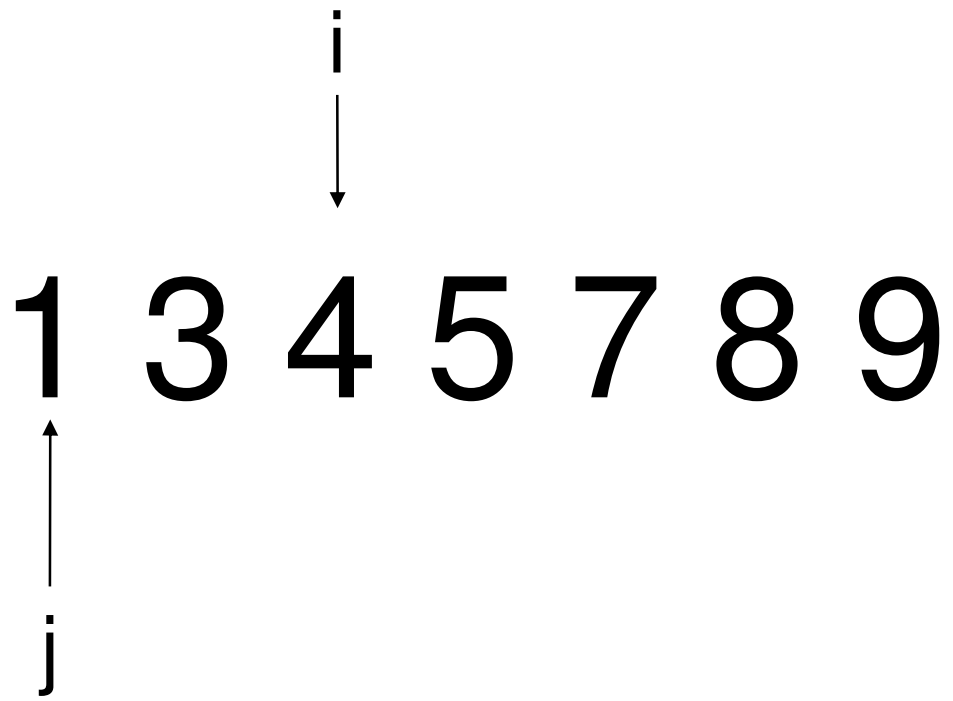

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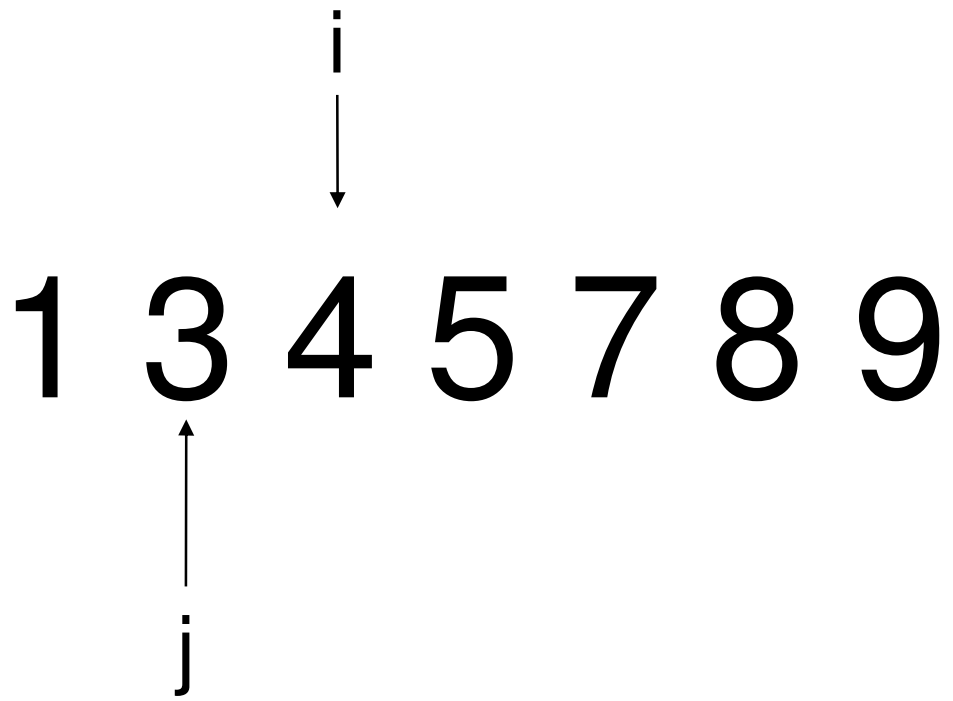


Insert Sort (melhoria)

1 3 4 5 7 8 9

Mergesort

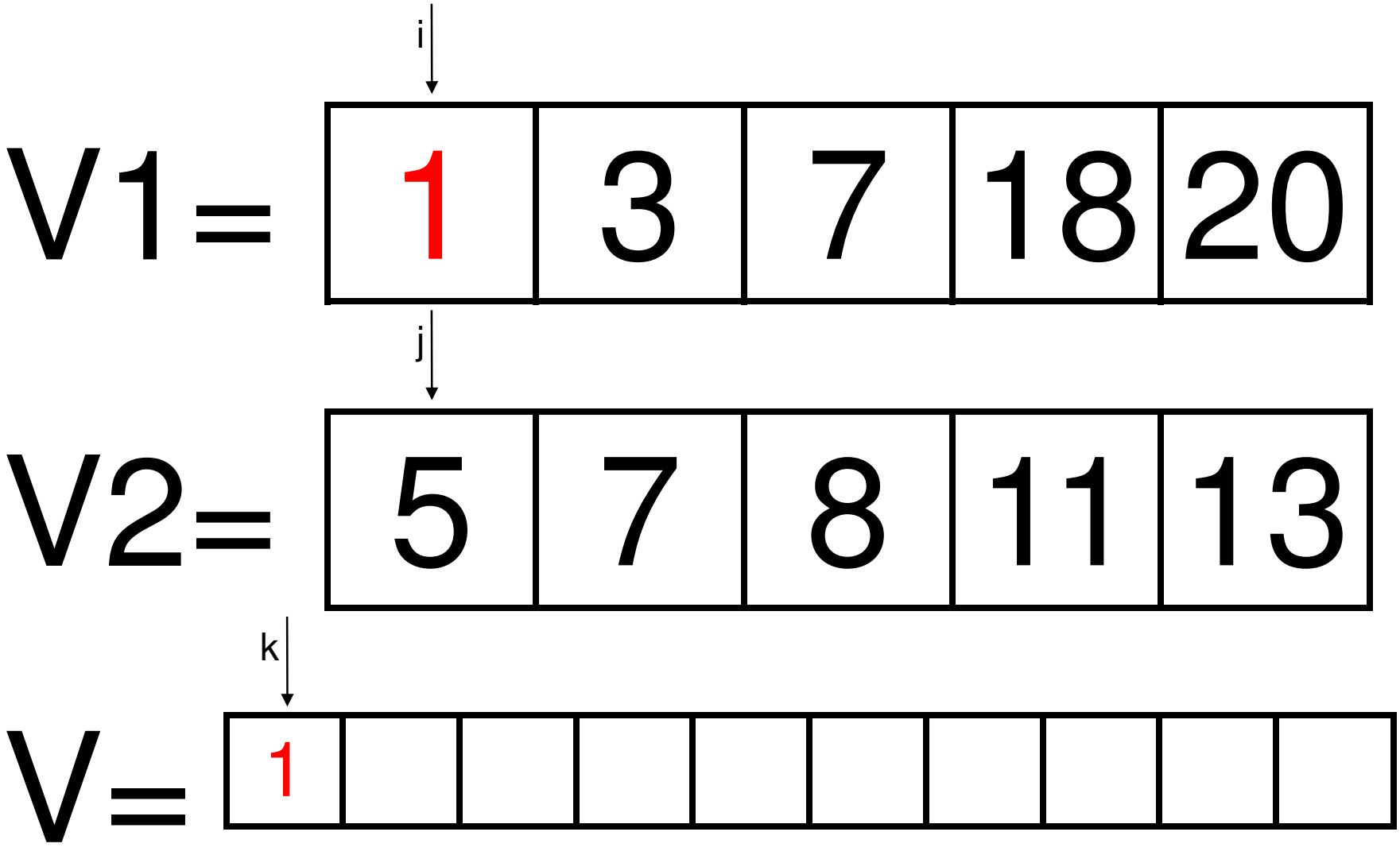
- Ordena inicialmente sub-vetores de tamanho 1 ou 2 e recursivamente dobra esse tamanho até ordenar o vetor total de tamanho N
- Baseia-se no merge de dois vetores ordenados

Merge de dois vetores ordenados

- Dados dois vetores V1 e V2, já ordenados, o **merge** cria um outro vetor V ordenado com os elementos de V1 e V2

```
local i,j,k = 1,1,1
for k=1,#v1+#v2 do
   if j>#v2 then v[k]=v1[i]; i=i+1
   else if i>#v1 then v[k]=v2[j]; j=j+1
   else if v1[i]<=v2[j] then v[k]=v1[i]; i=i+1
   else v[k]=v2[j]; j=j+1 end
end
```

Merge de dois vetores



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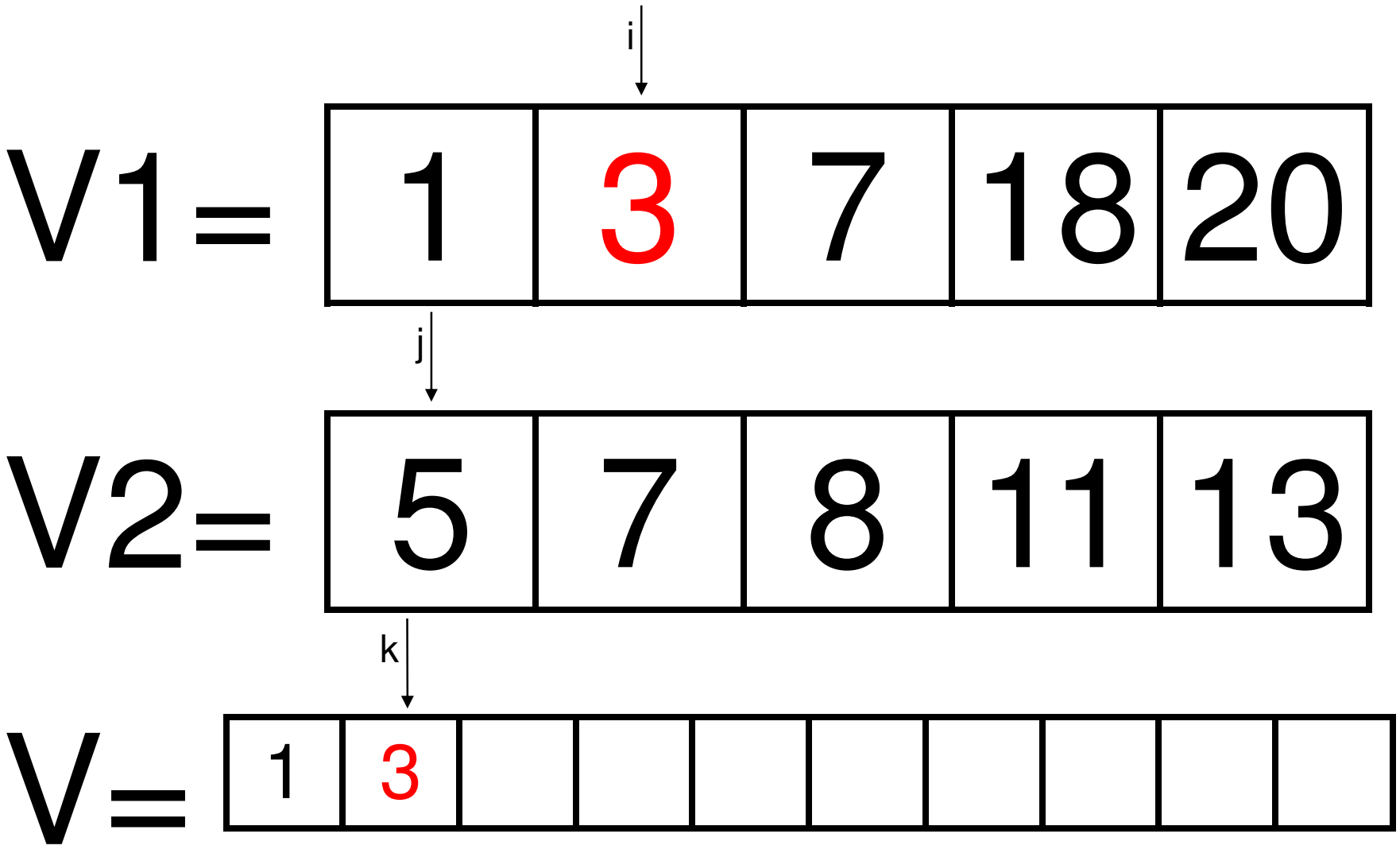

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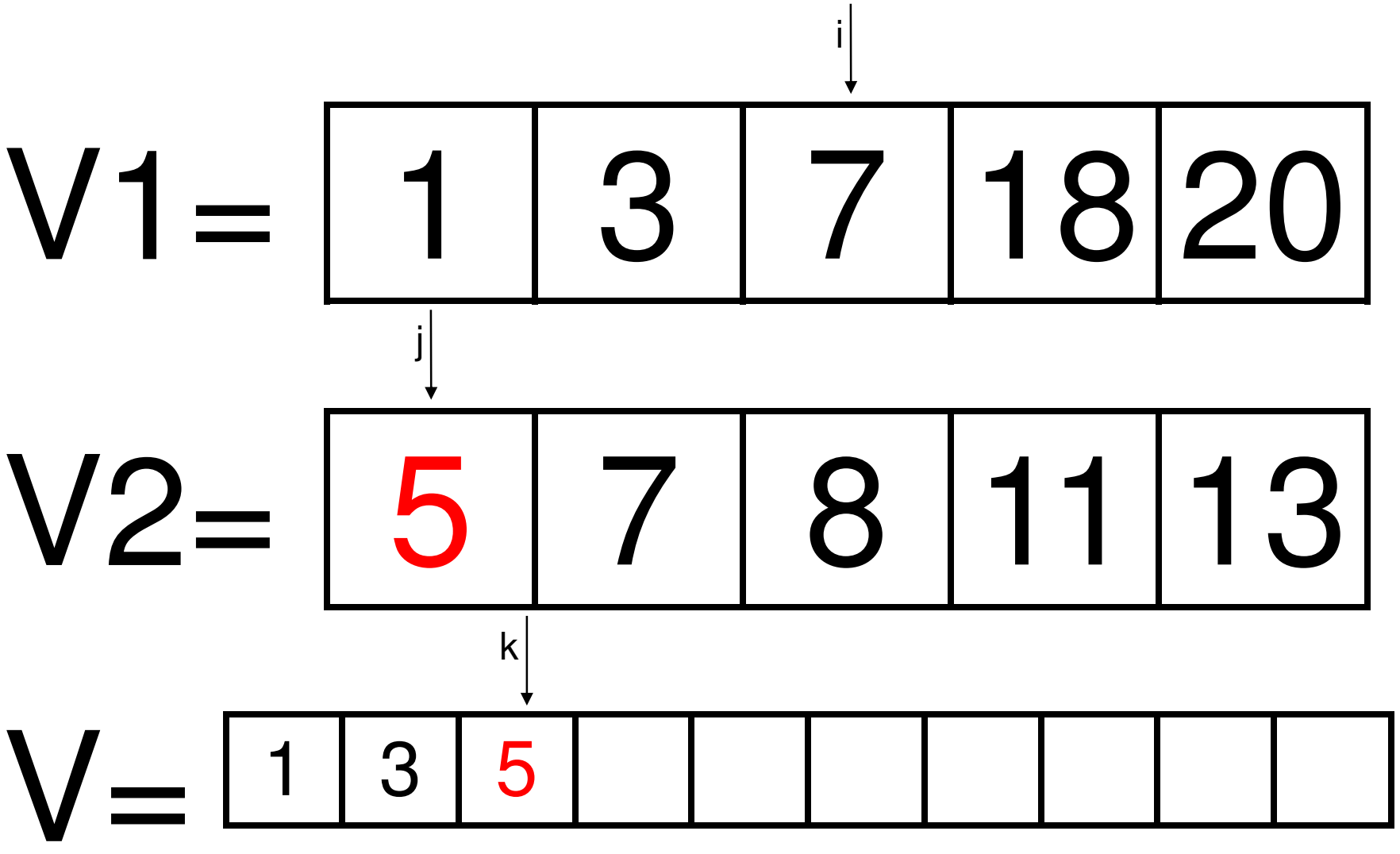

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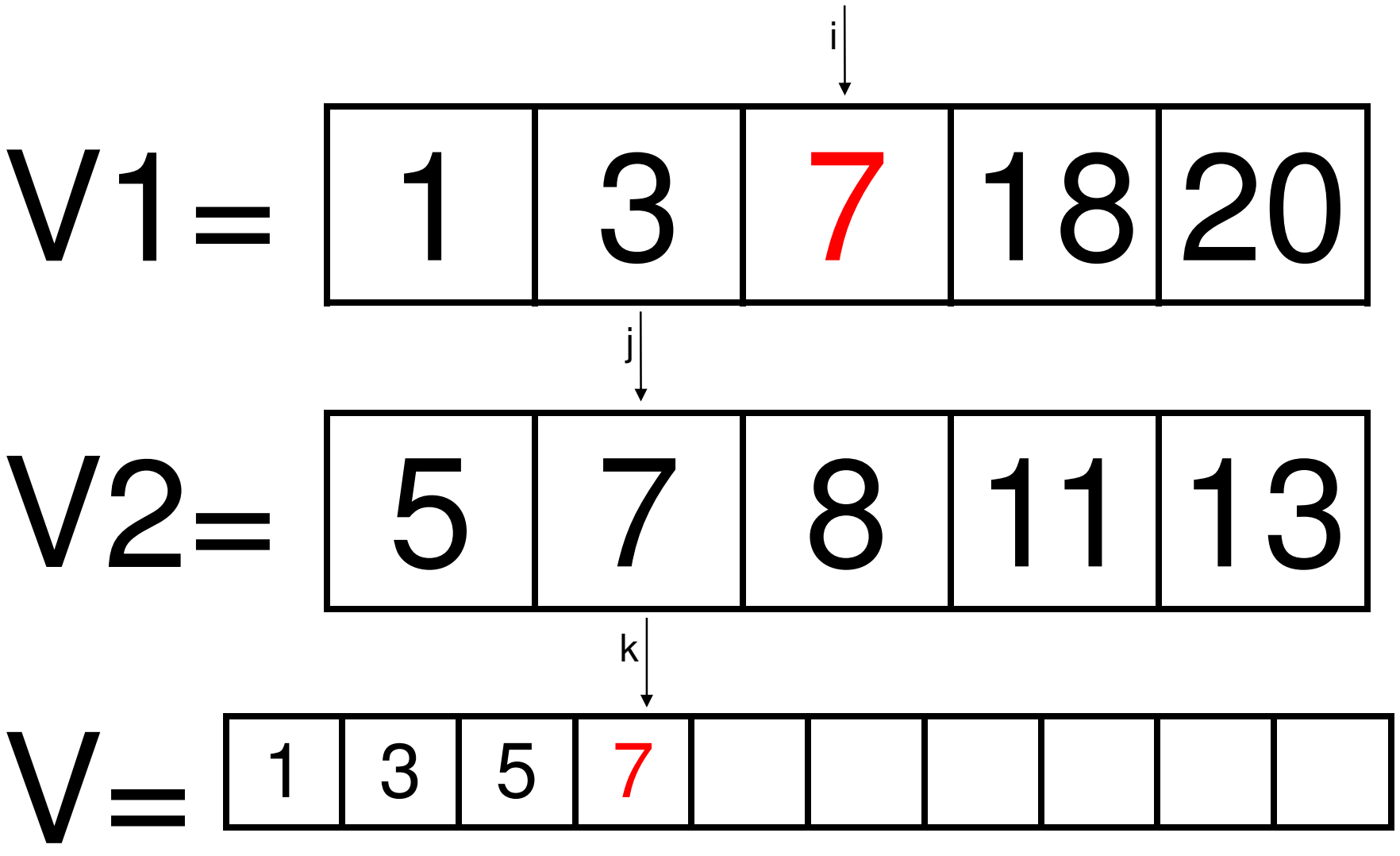

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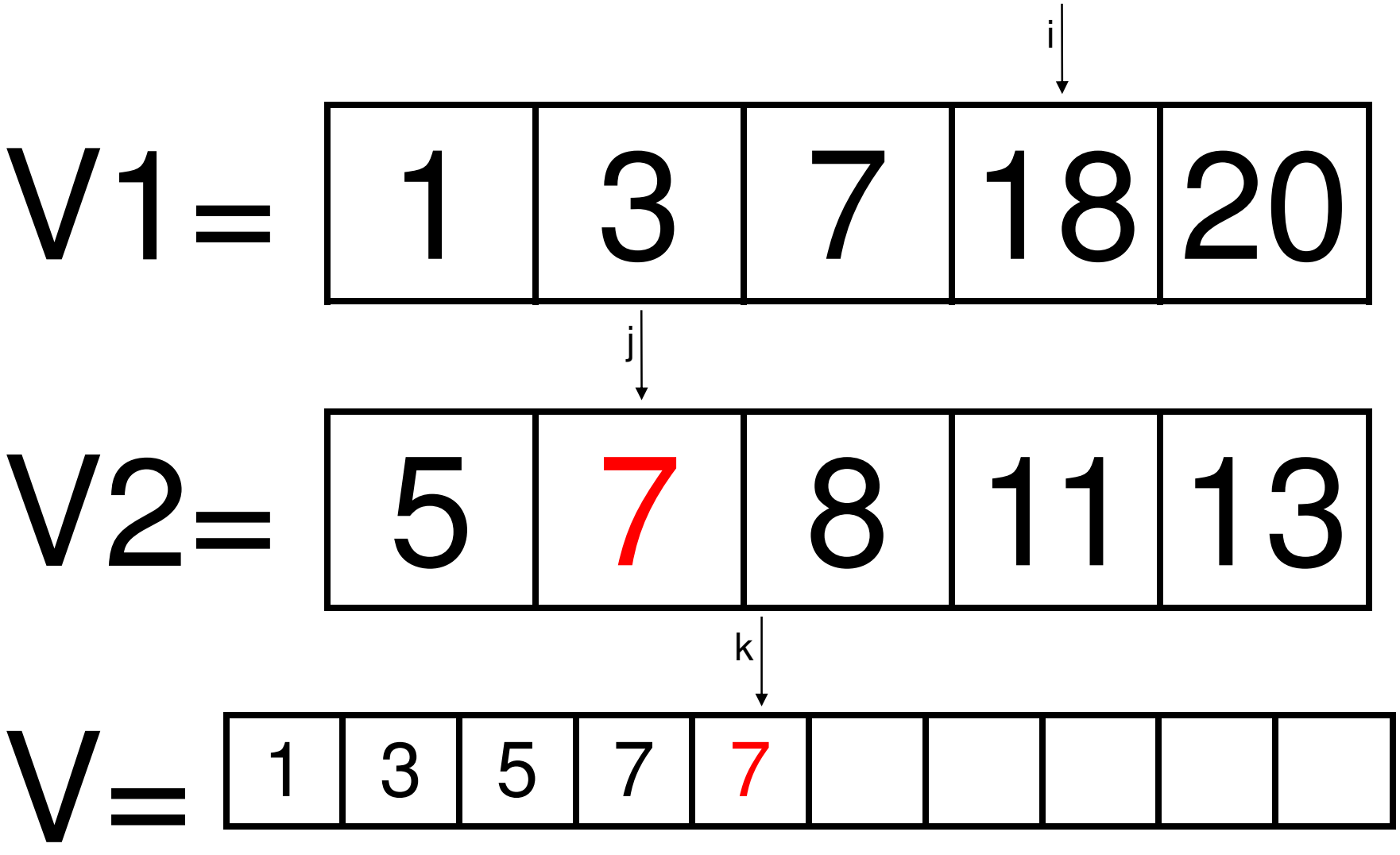

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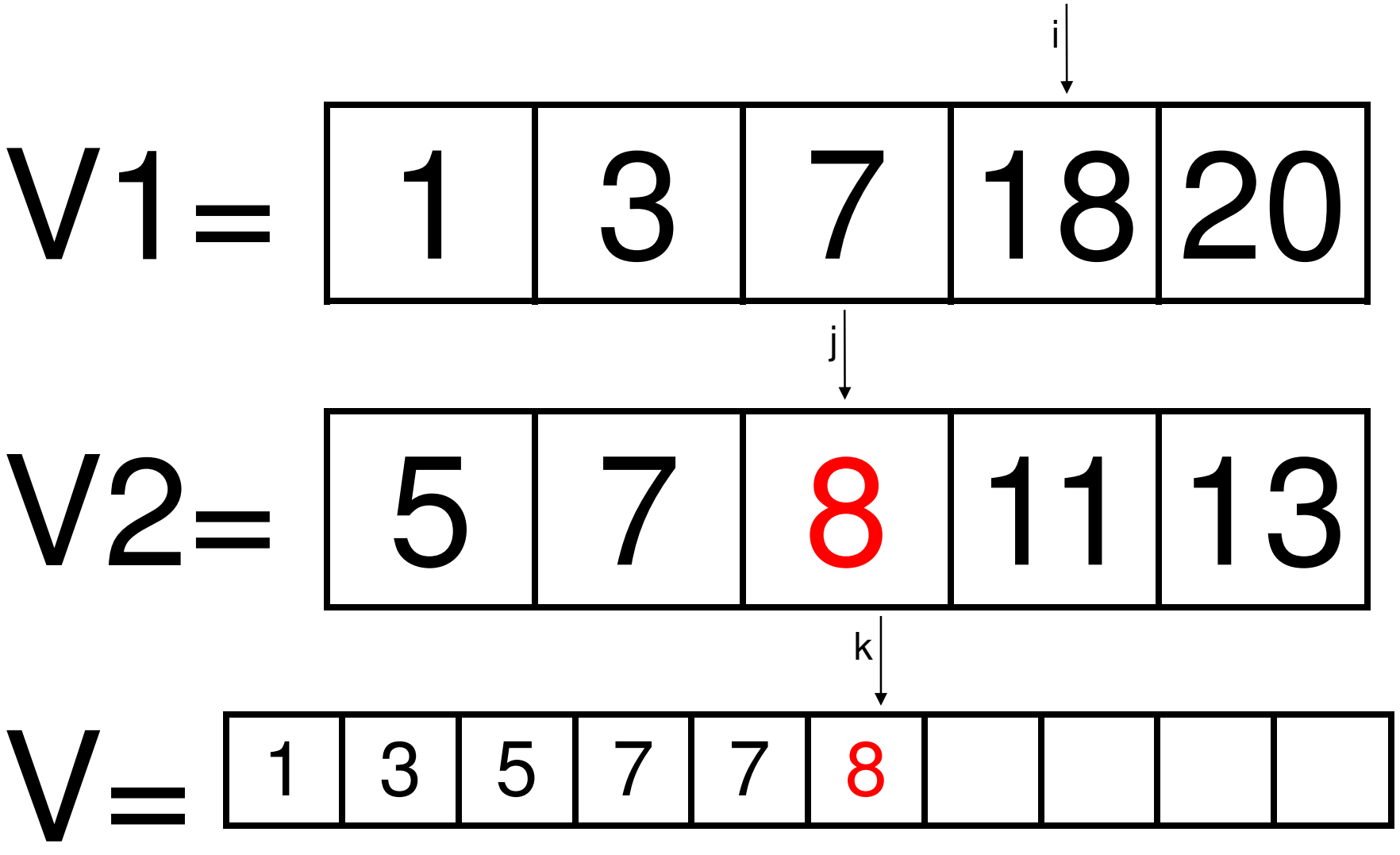

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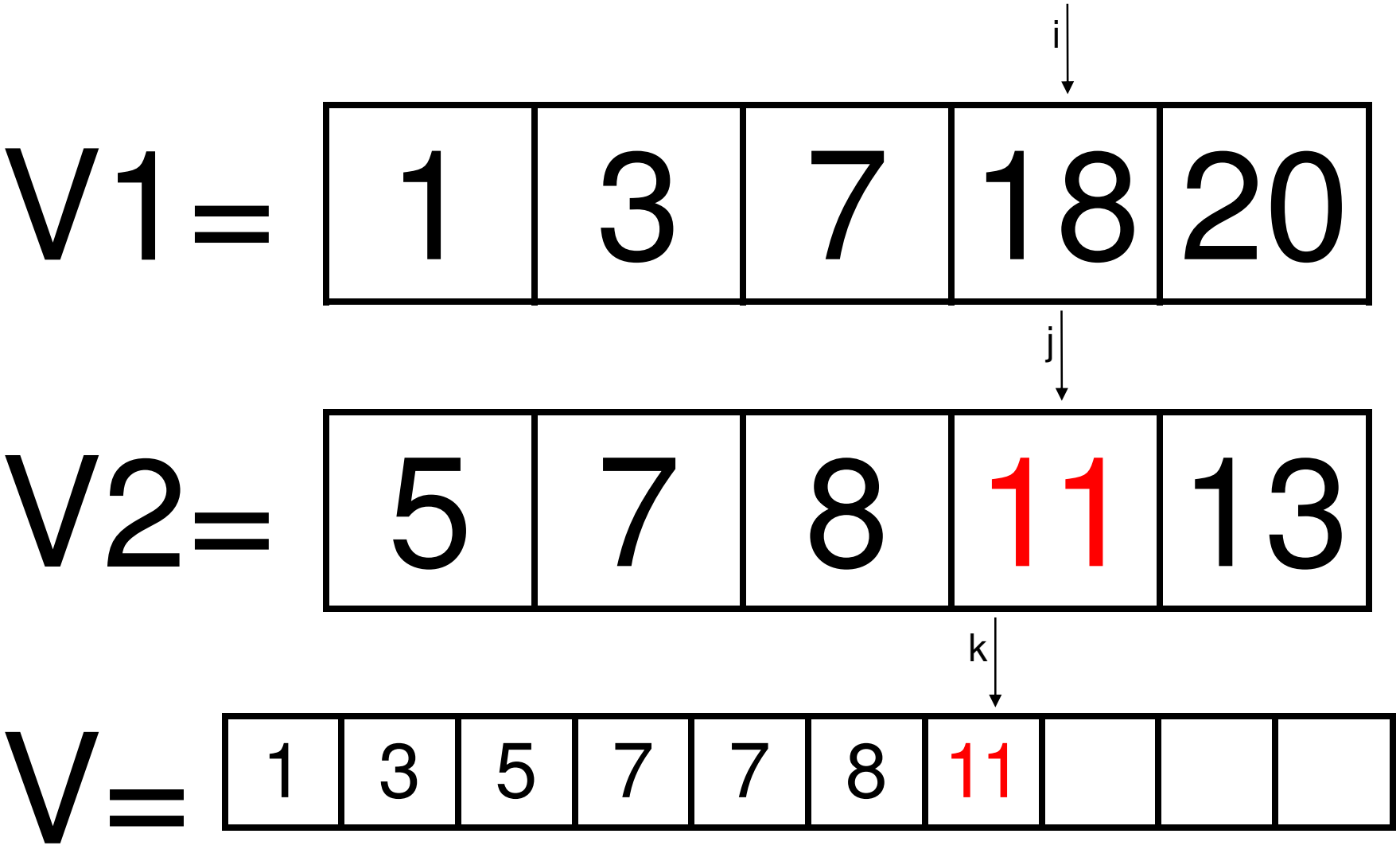

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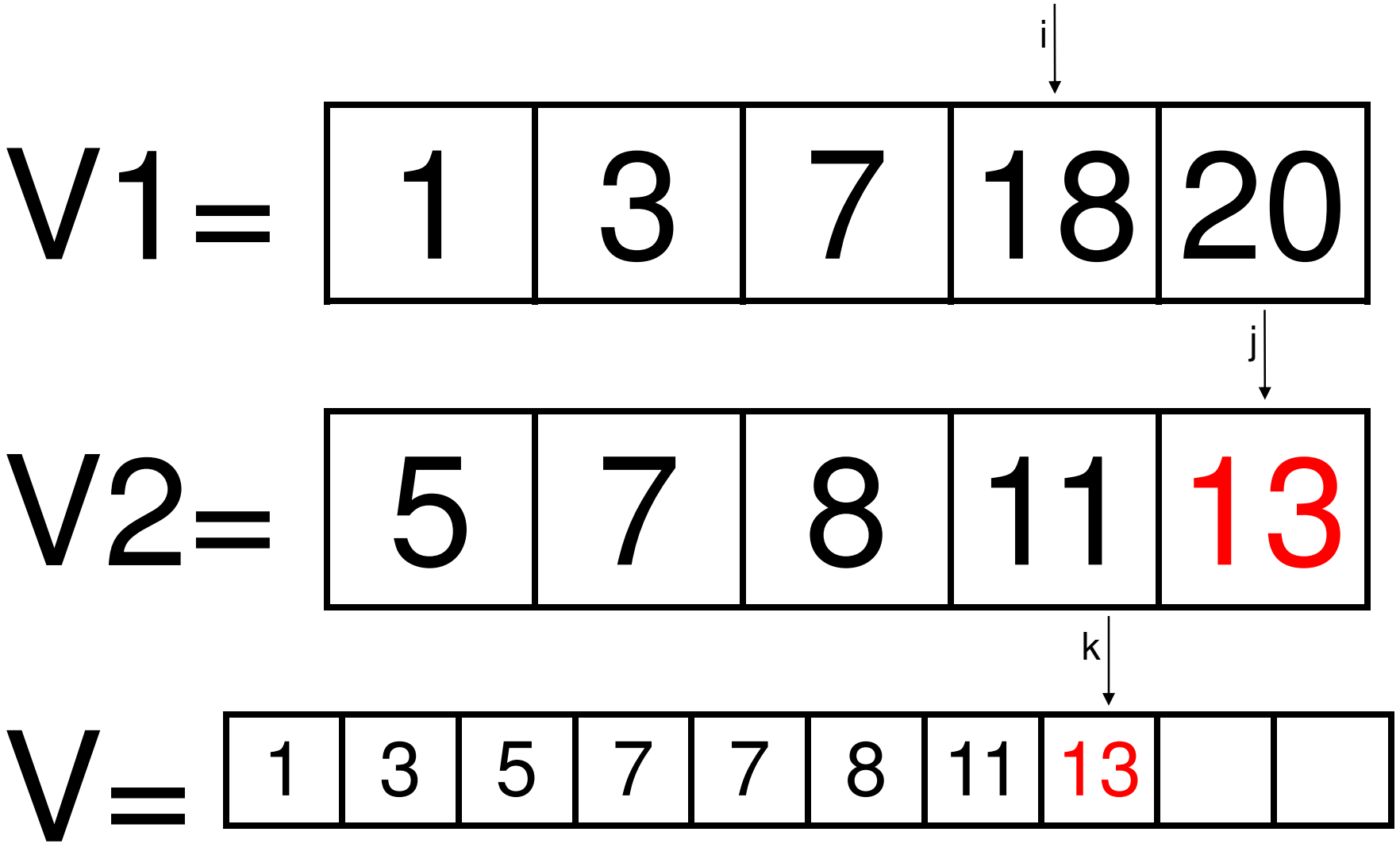

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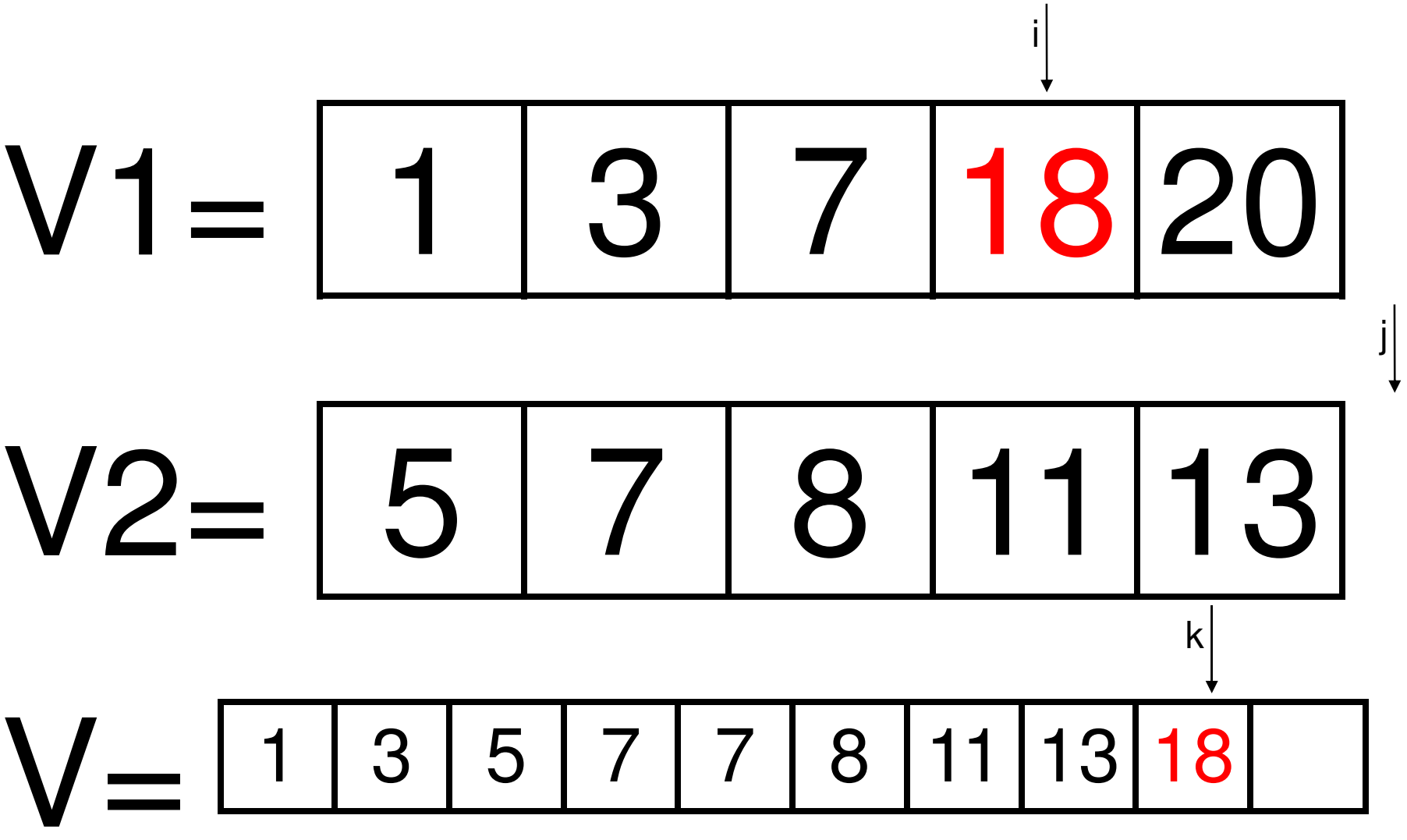

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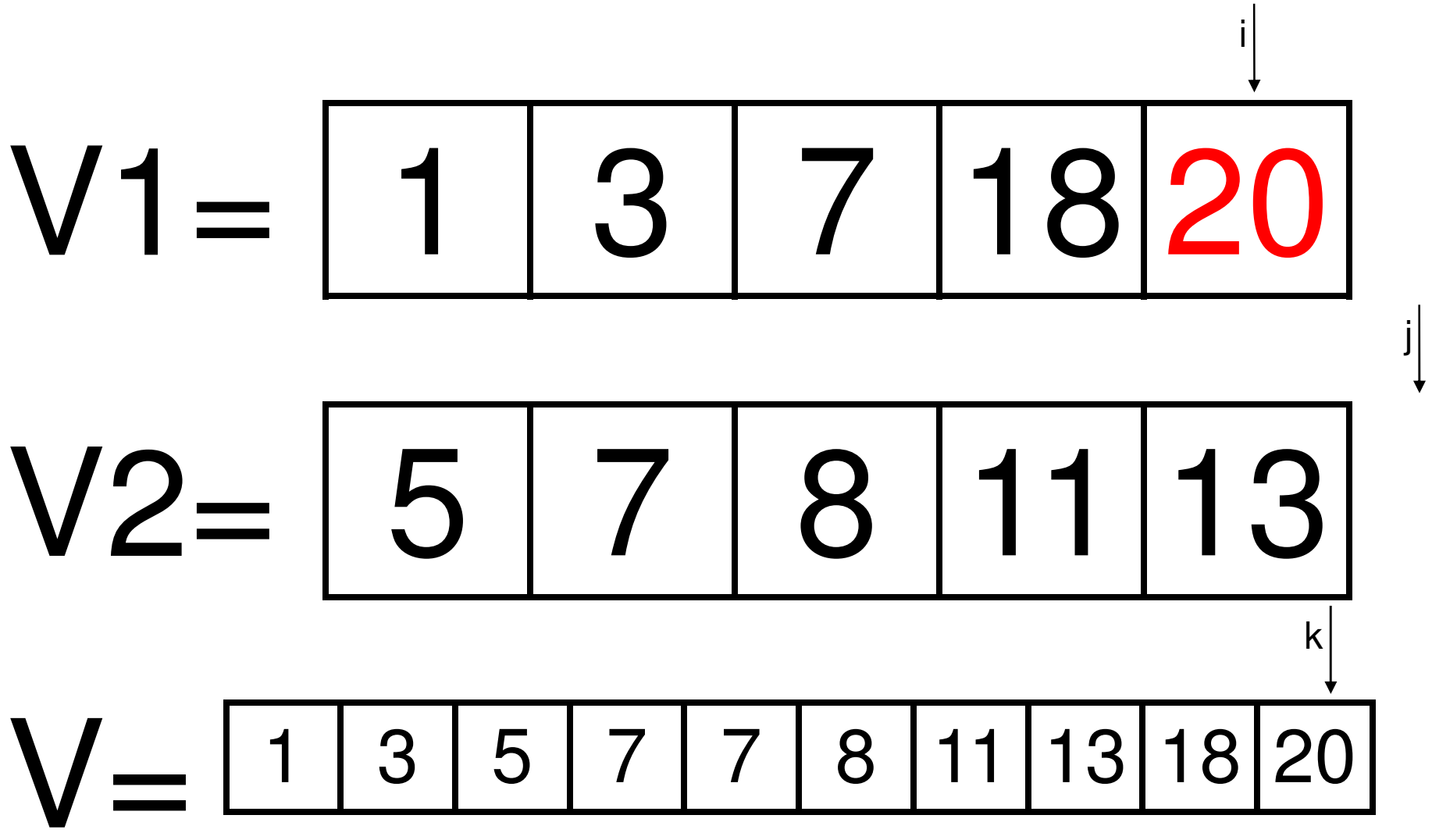

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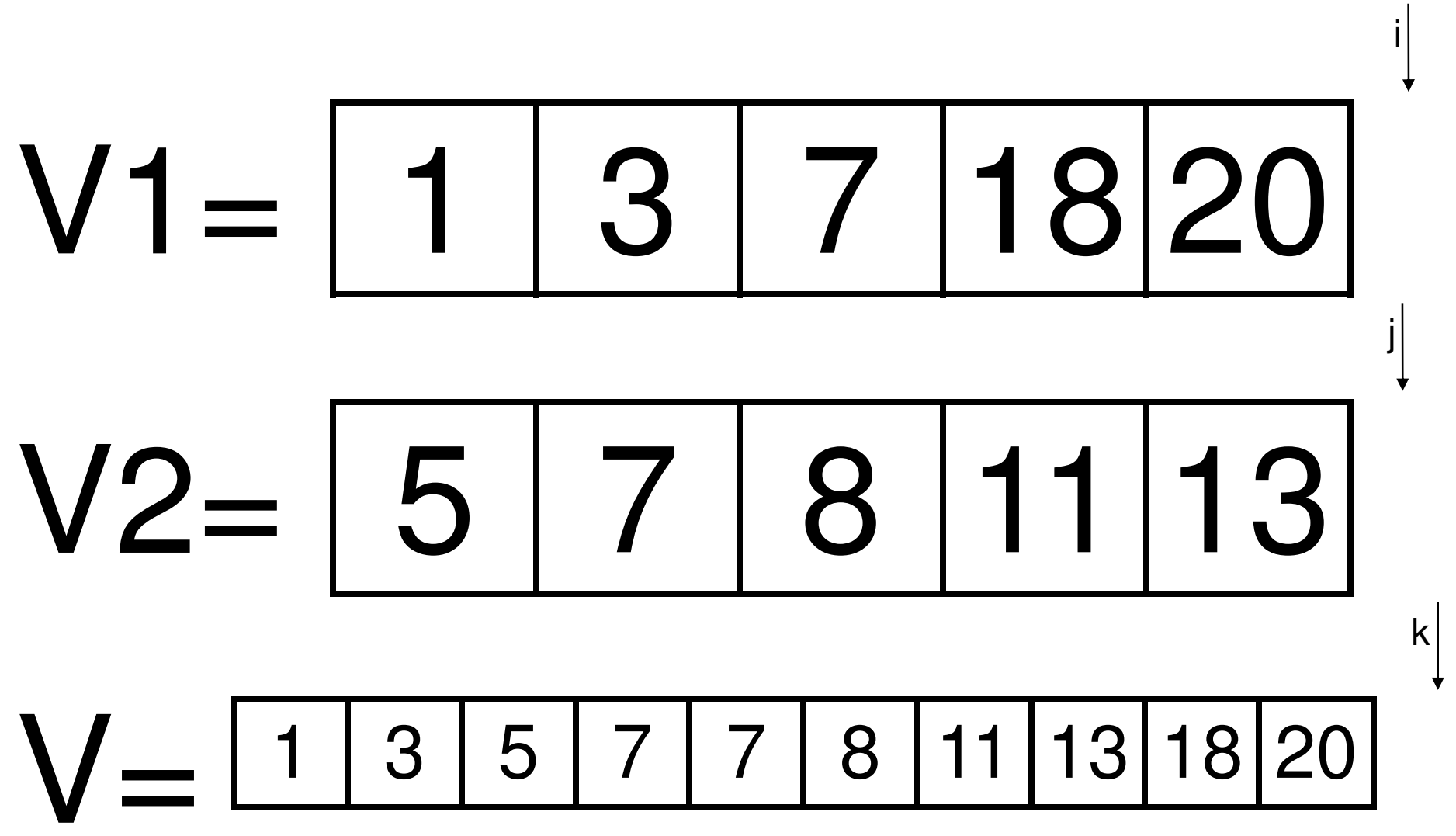
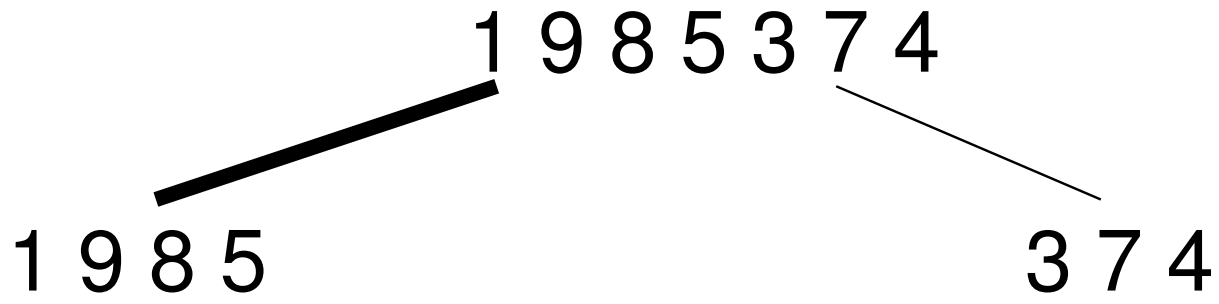


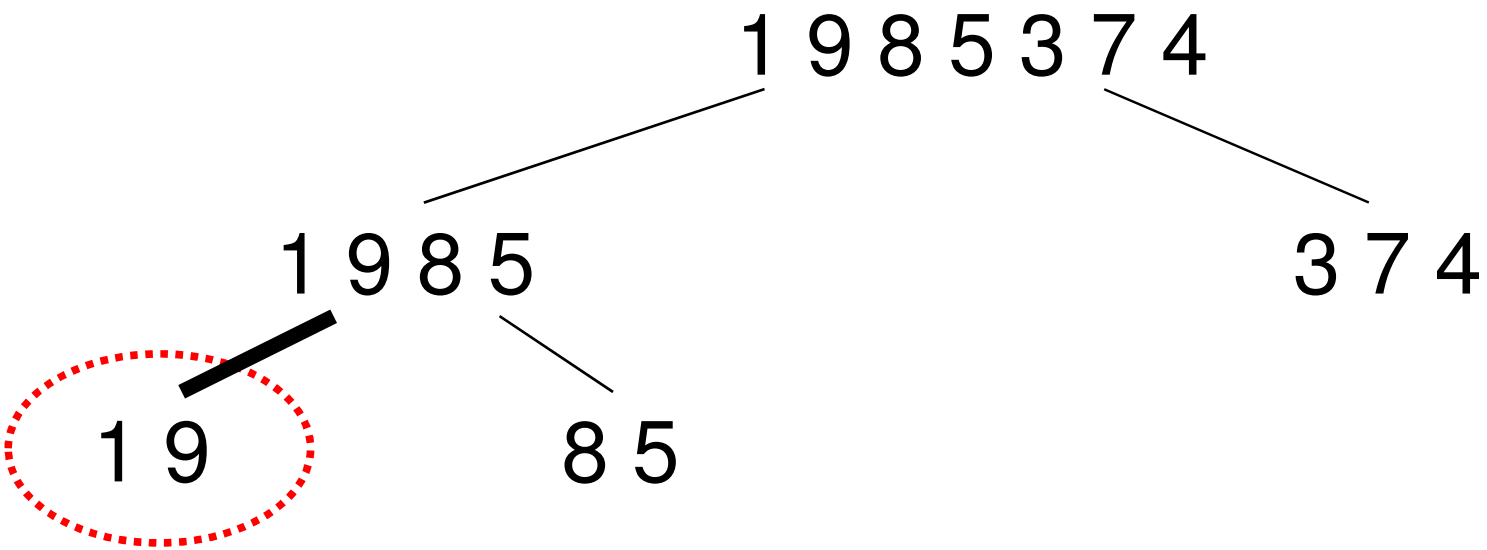
Mergesort

1 9 8 5 3 7 4

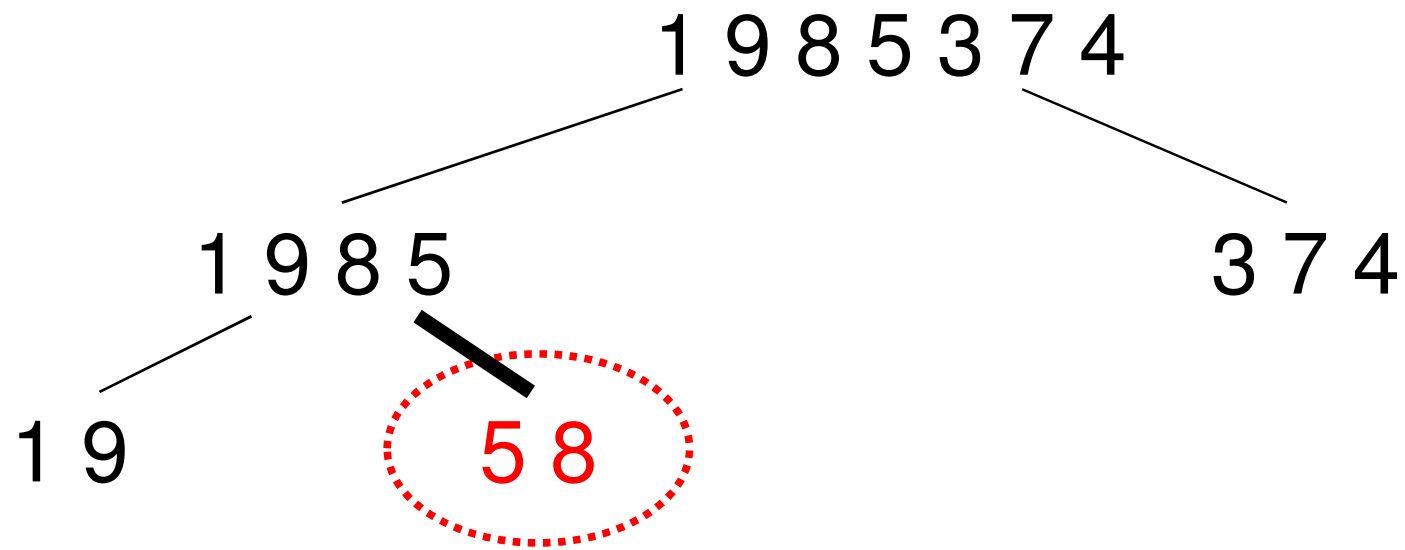
Mergesort



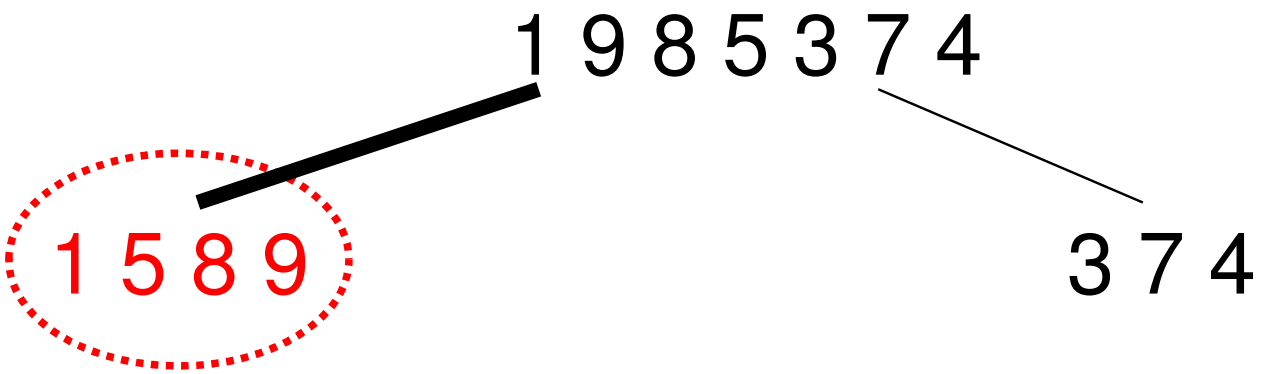
Mergesort



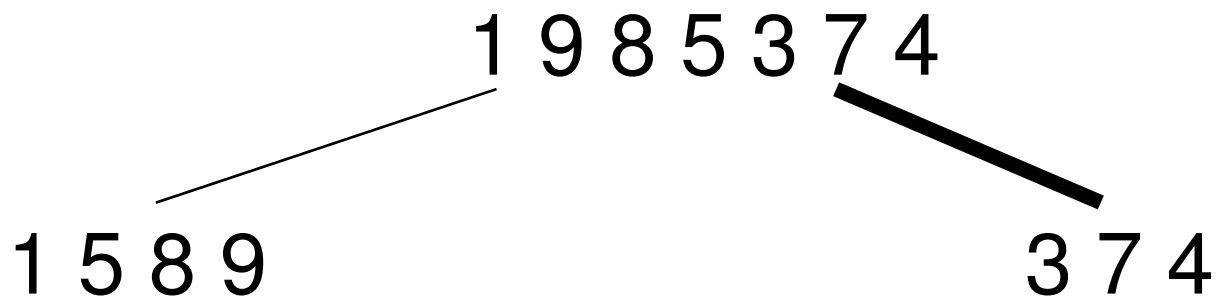
Mergesort



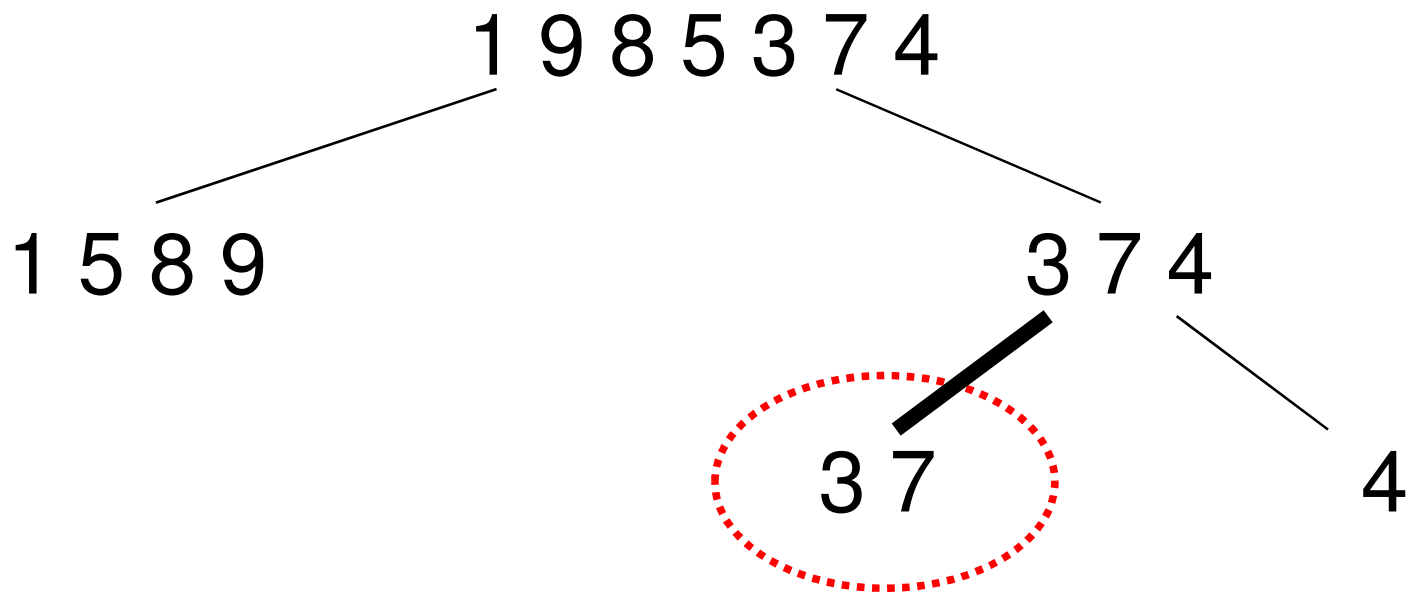
Mergesort



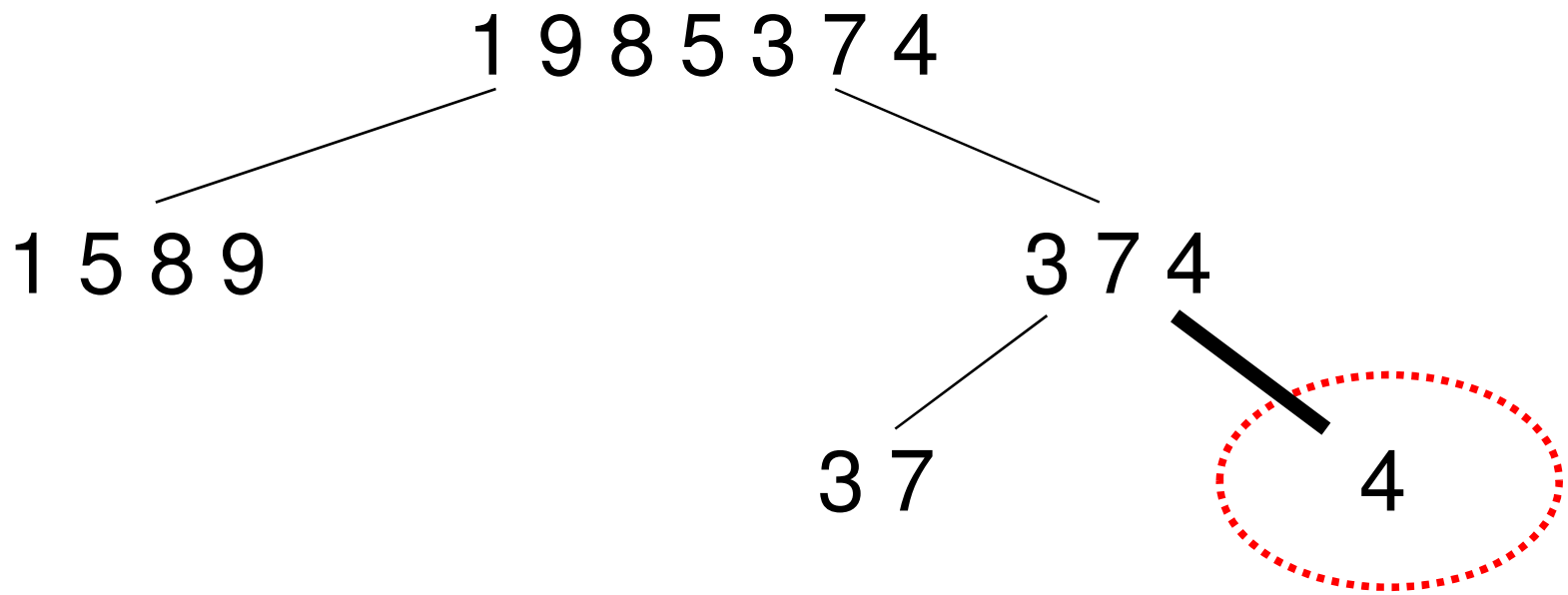
Mergesort



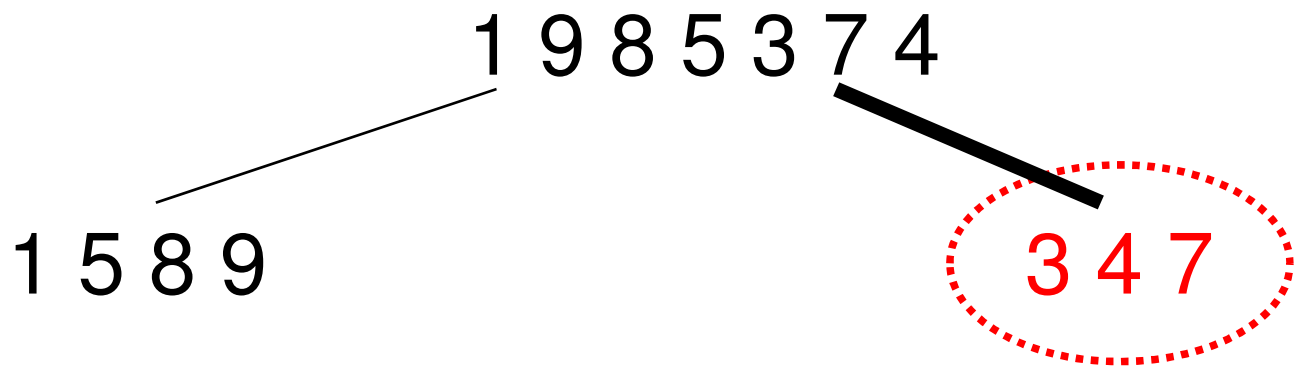
Mergesort



Mergesort



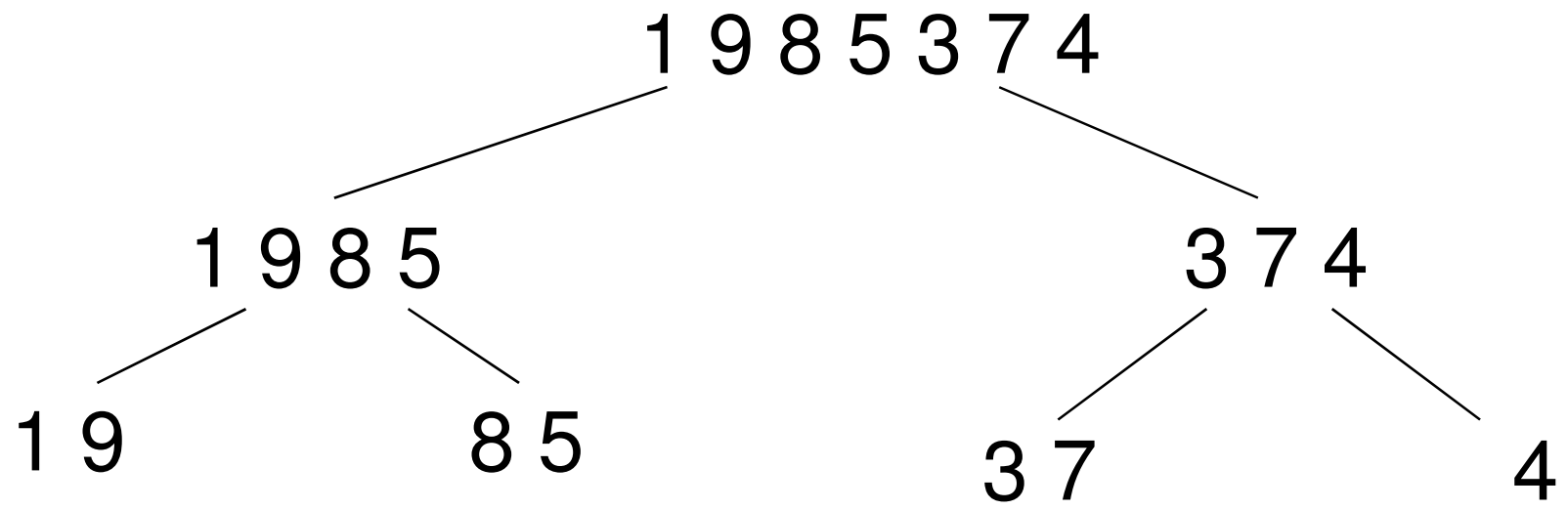
Mergesort



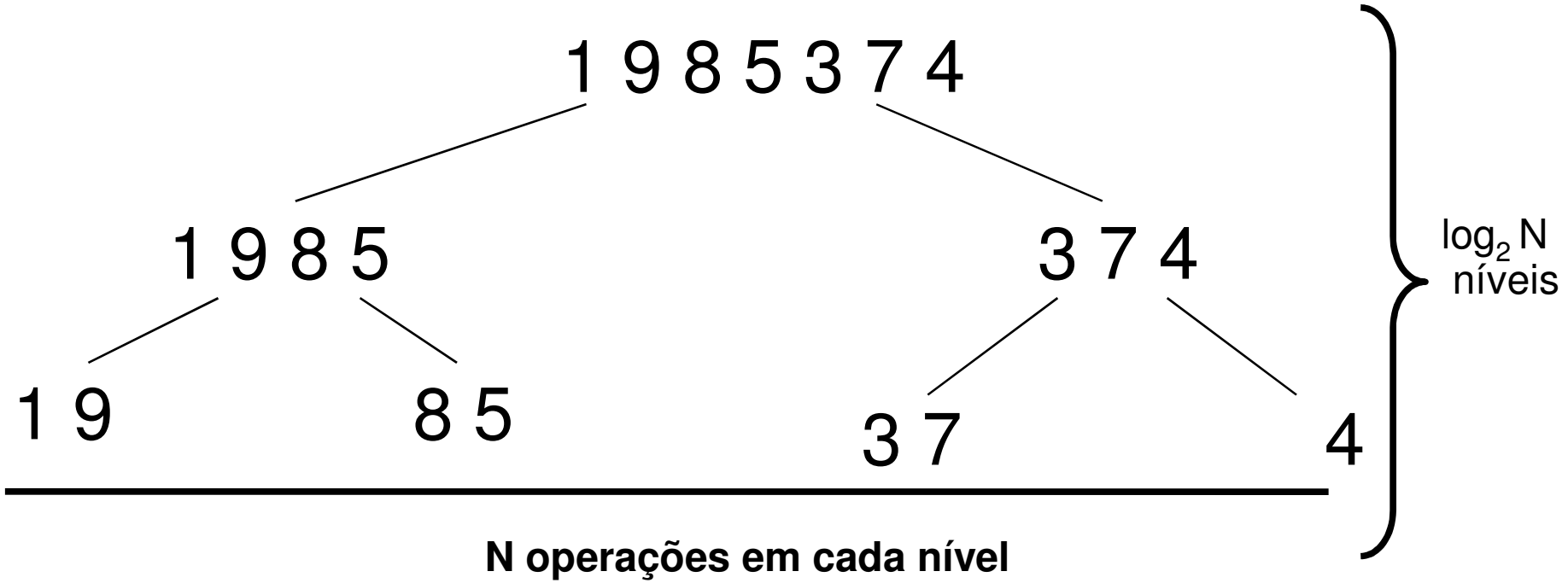
Mergesort

1 3 4 5 7 8 9

Mergesort - operações



Mergesort - operações



Total de operações:

$N * \log_2 N$

Heapsort

- Ordenação baseada na estrutura de dados *heap*
  - A *heap* é um vetor que pode ser visto como uma árvore binária completa
  - Cada nó da árvore equivale a um valor no vetor

Primitivas da Heap

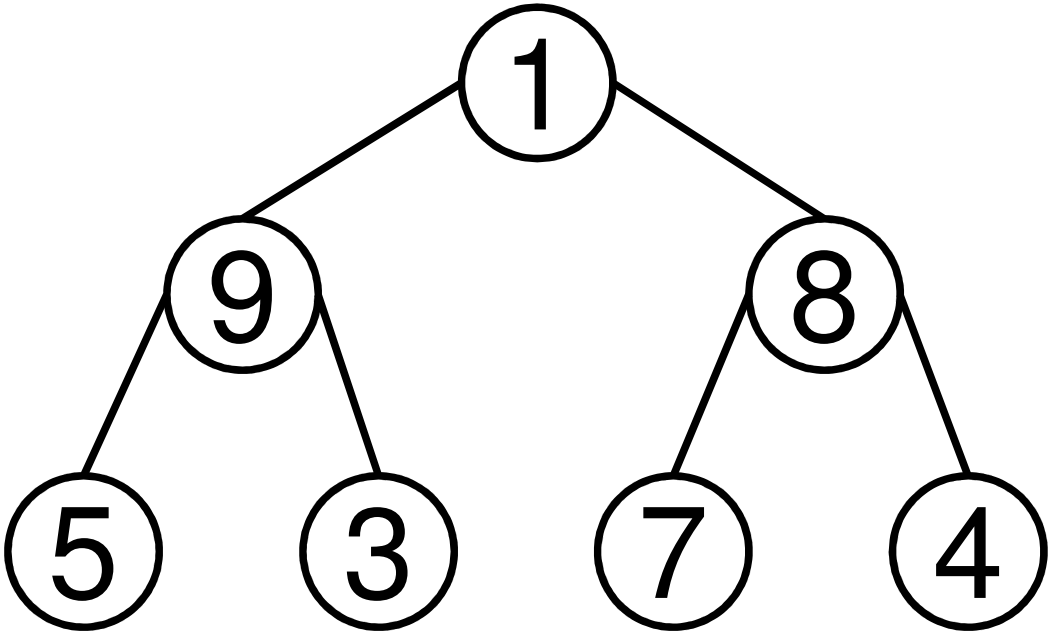
- length(V)
  - número de elementos no vetor V
- heap-size(V)
  - número de elementos da *heap* armazenados no vetor V

Vetor Heap – primitivas de árvore

- parent(i)
  - return i/2
- left(i)
  - return 2i
- right(i)
  - return 2i+1

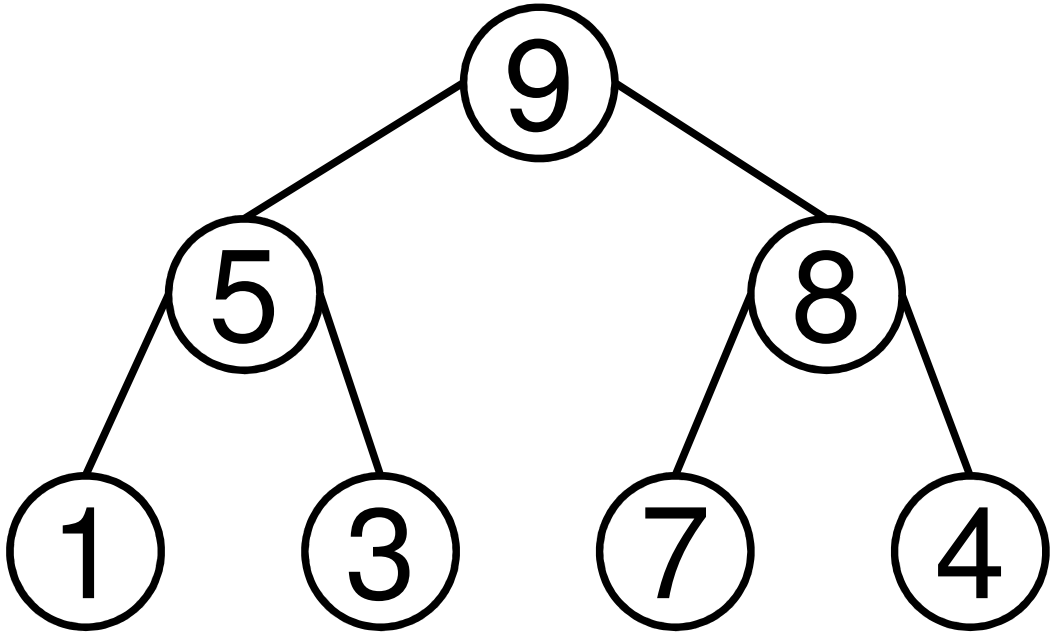
Heap como um vetor

1	9	8	5	3	7	4



Propriedade da Heap

- Para cada nó i que não seja a raiz, então:
  - V[parent(i)] >= V[i]
- Exemplo:



LAWS

Primitiva Heapify

- Função para manter a propriedade da heap

```
function heapify(V, i)
    l = left(i)
    r = right(i)
    if l<=heap-size(V) and V[l]>V[i] then largest = l
      else largest = i end
    if r<=heap-size(V) and V[r]>V[largest]
      then largest=r end
    if largest != i then
         V[i], V[largest] = V[largest], V[i]
         heapify(V, largest)
      end
  end
```

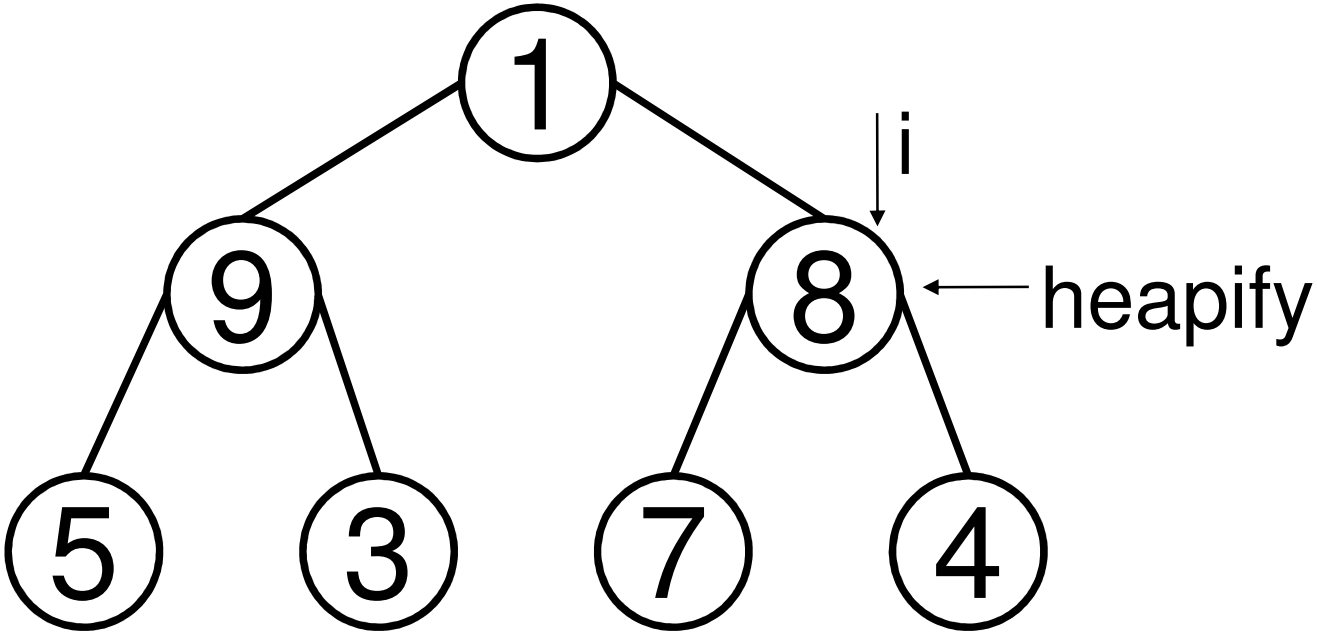
Construindo uma Heap

- Primitiva que constrói uma *heap* a partir de um vetor de N elementos

```
function build_heap(V)
    heap-size(V) = length(V) -- #V
    for i=length(V)/2,1,-1 do
        heapify(V,i)
    end
end
```

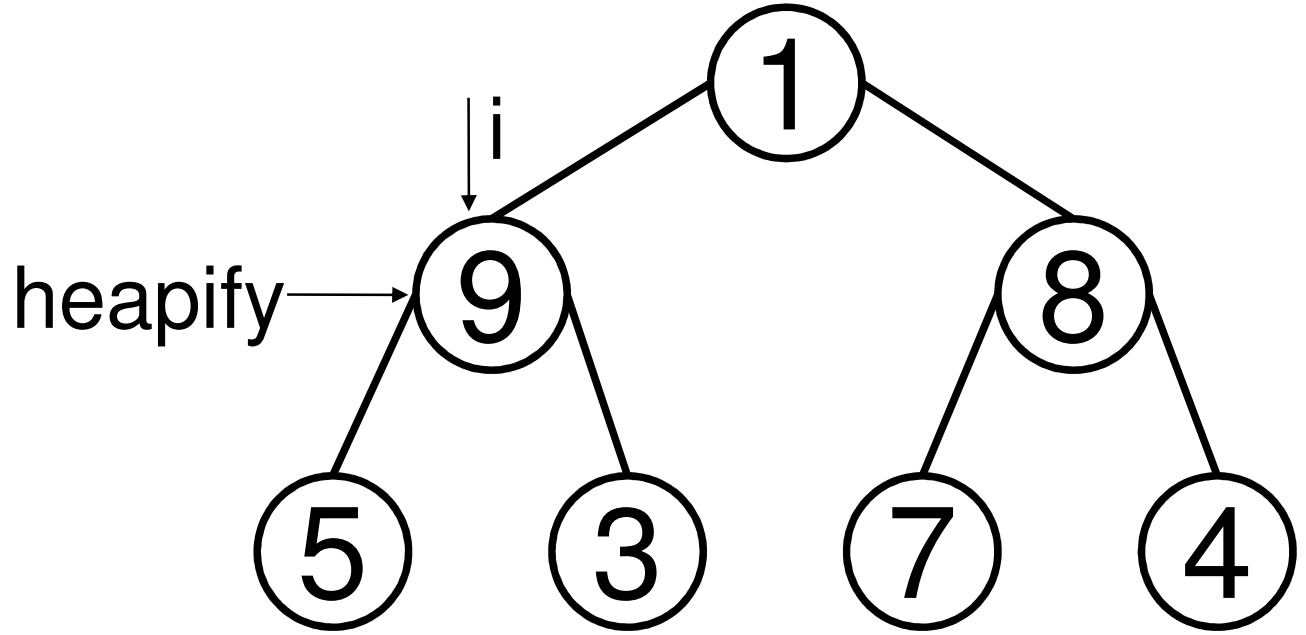
Construindo uma Heap

1	9	8	5	3	7	4



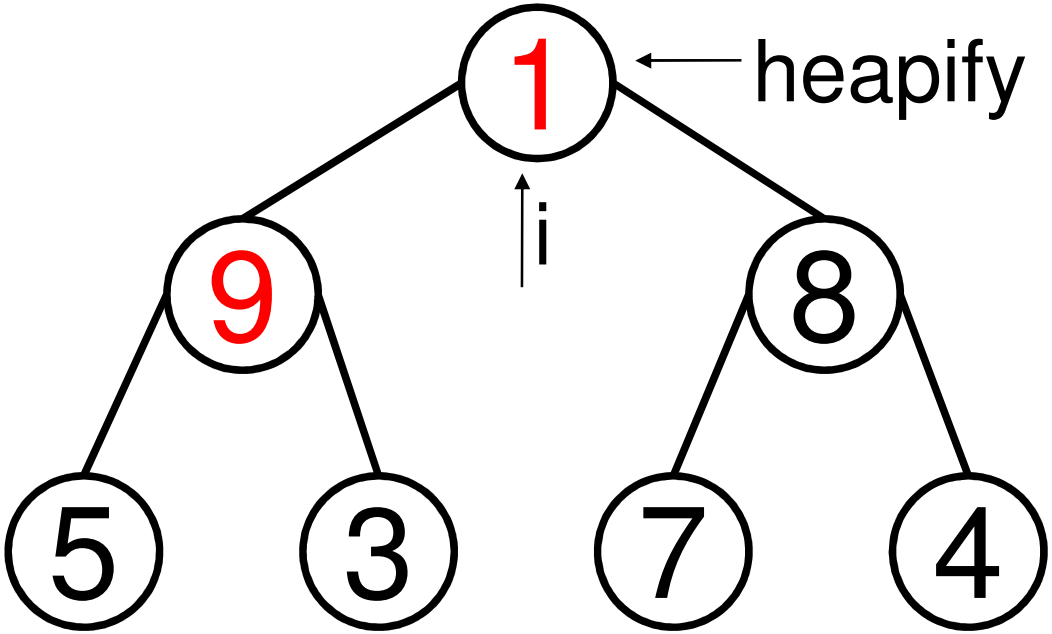
Construindo uma Heap

1	9	8	5	3	7	4



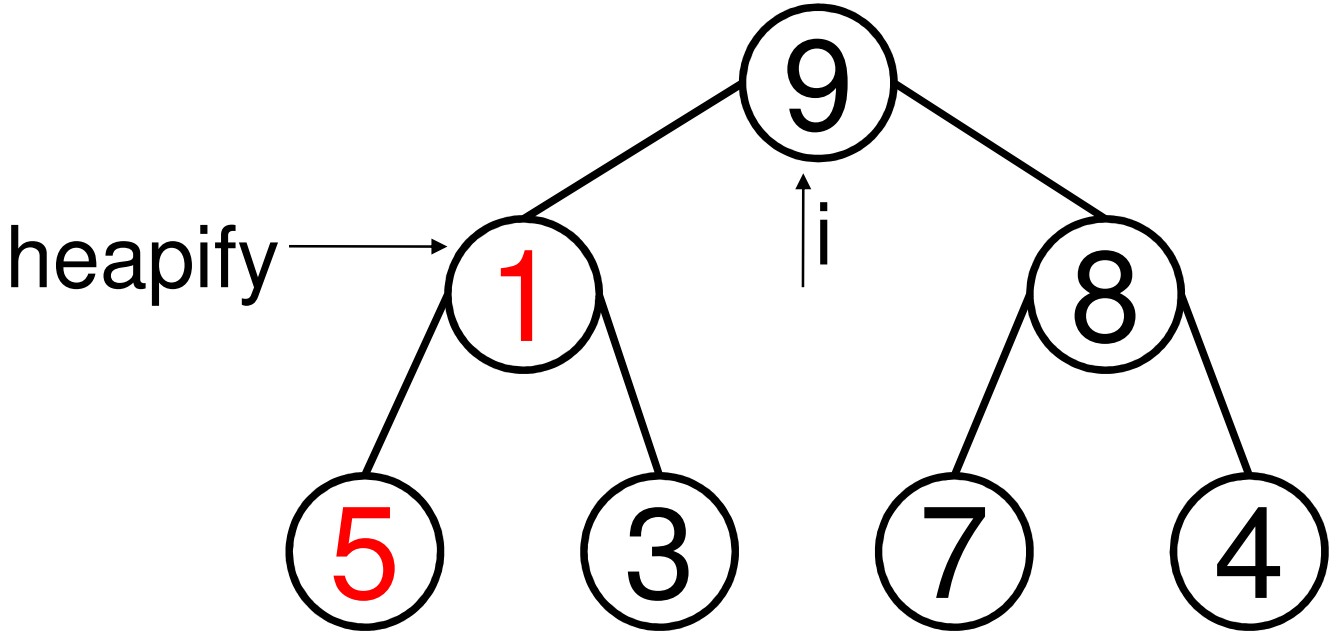
Construindo uma Heap

1	9	8	5	3	7	4



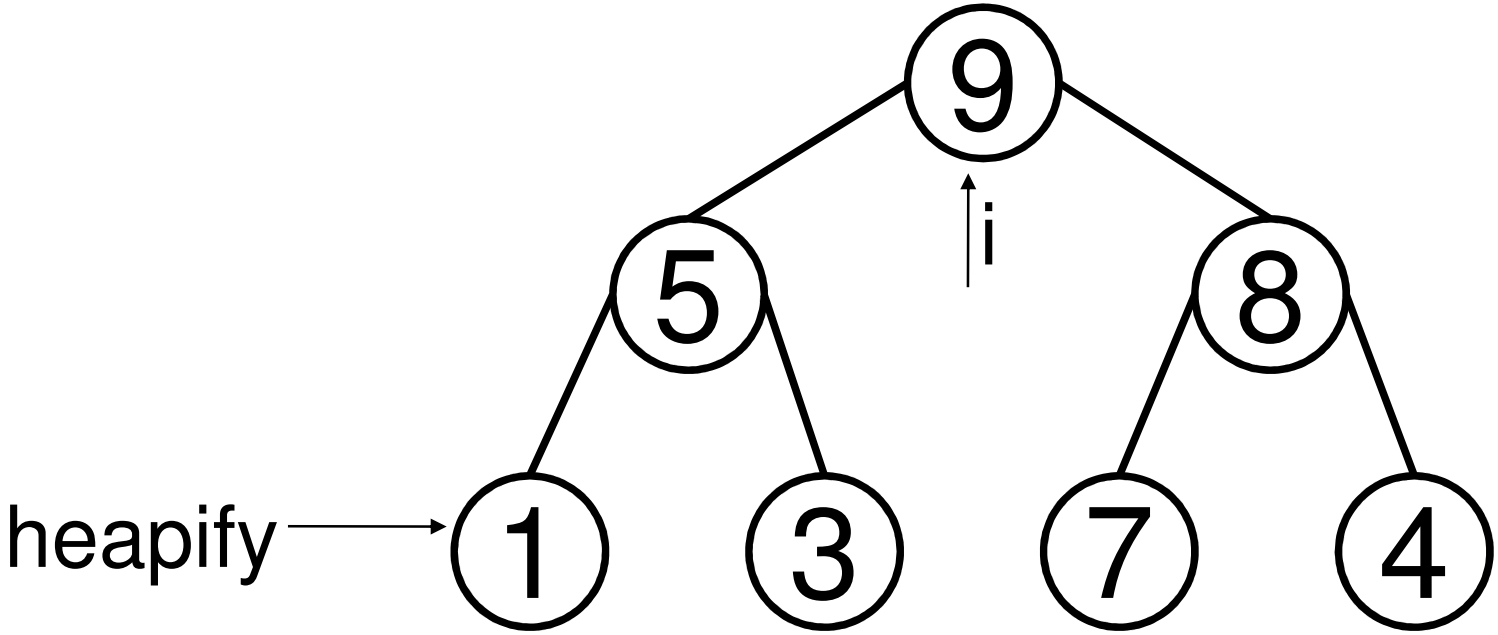
Construindo uma Heap

9	1	8	5	3	7	4



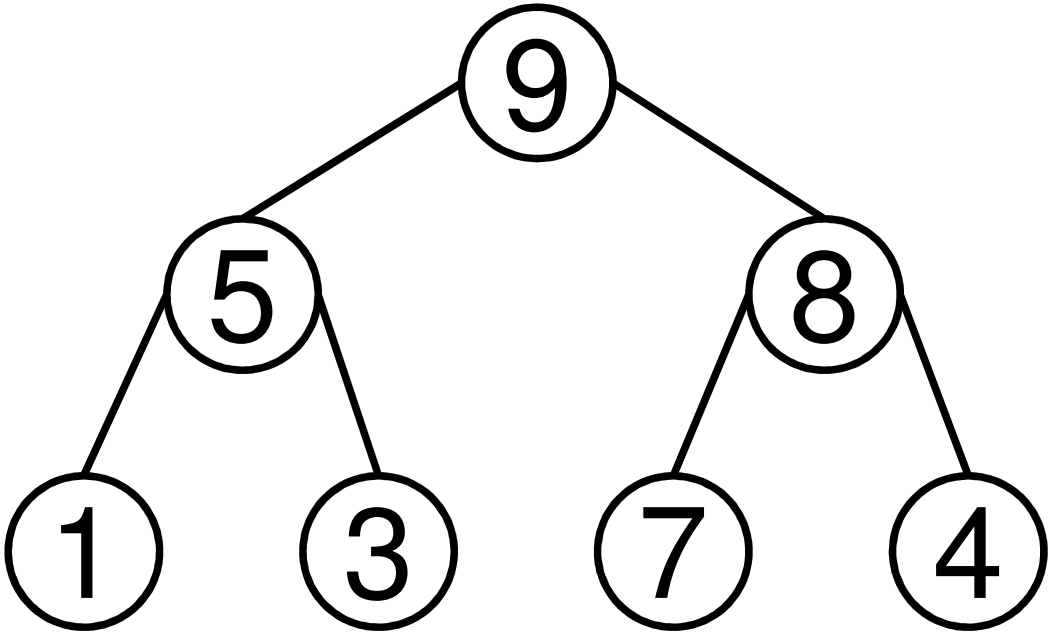
Construindo uma Heap

9	5	8	1	3	7	4



Construindo uma Heap

9	5	8	1	3	7	4

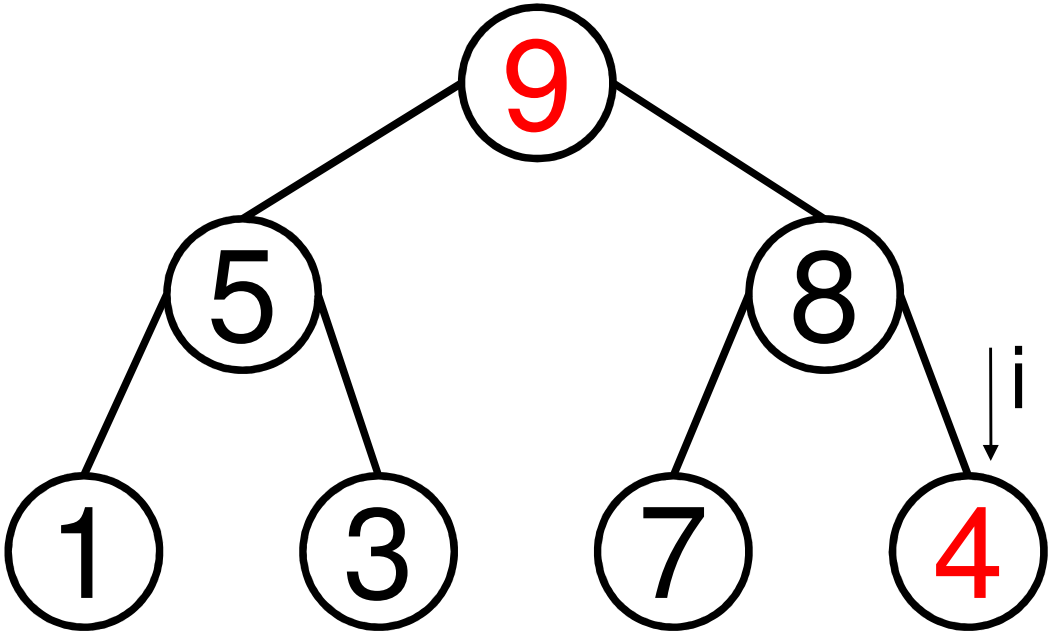


Heapsort

```
function heapsort(V)
    for i=length(V),2,-1 do
        V[1], V[i] = V[i], V[1]
        heap-size(V) = heap-size(V)-
  1
        heapify(V,1)
    end
  end
```

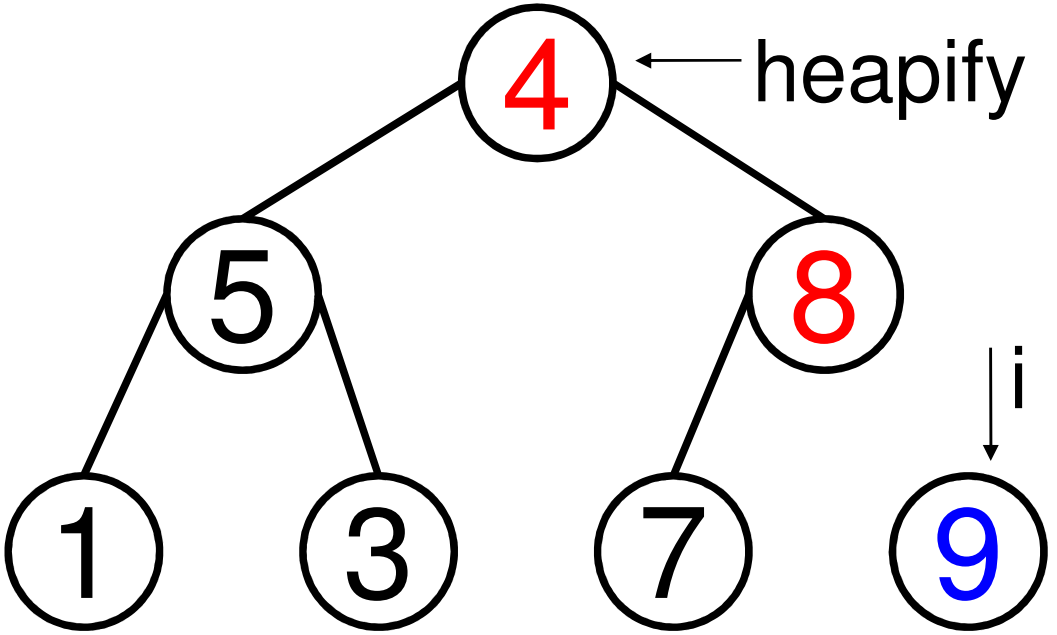
Heapsort

9	5	8	1	3	7	4



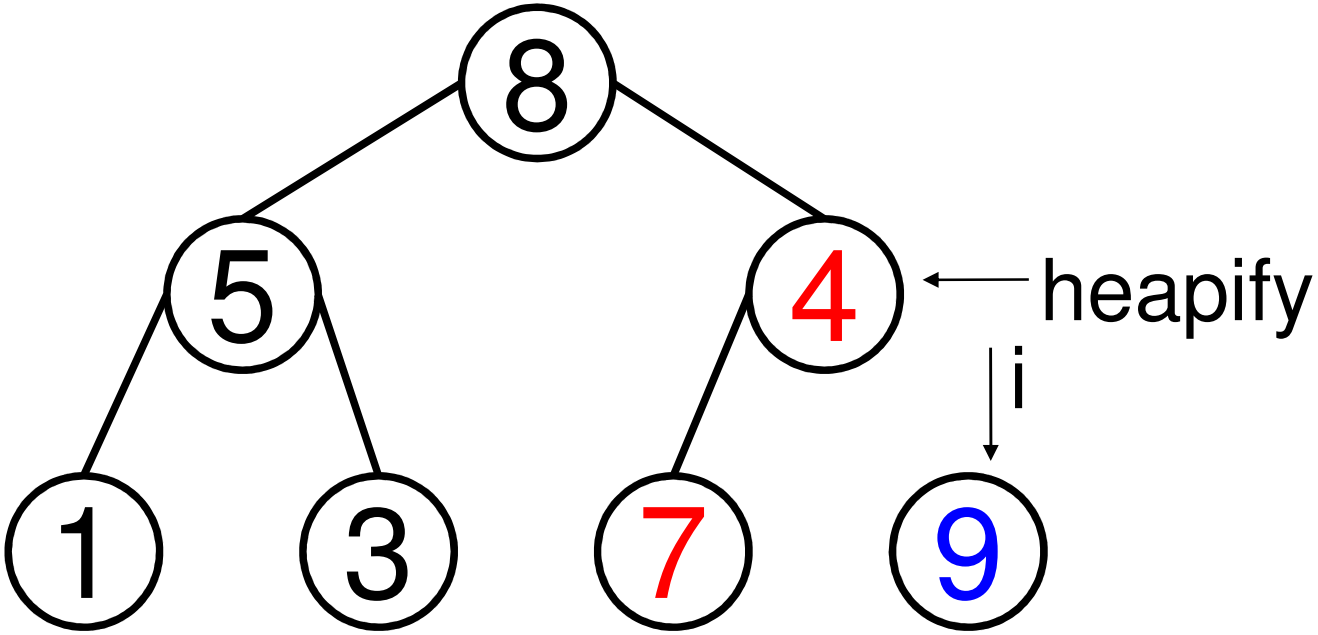
Heapsort

4	5	8	1	3	7	9



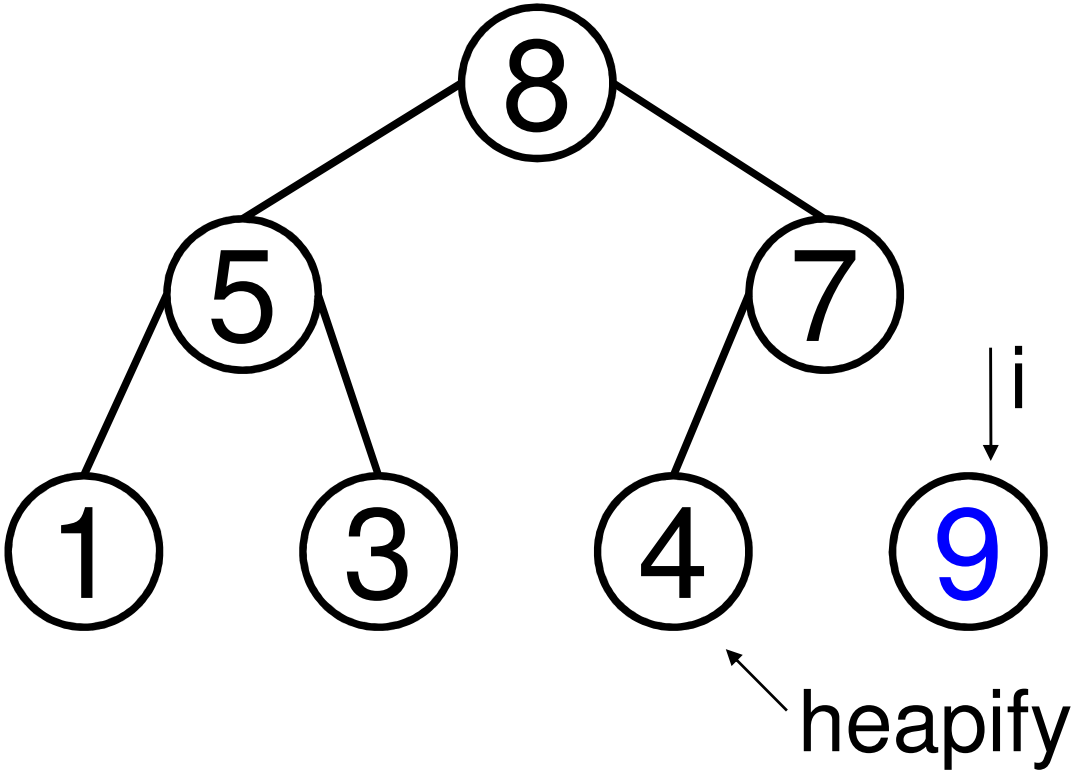
Heapsort

8	5	4	1	3	7	9



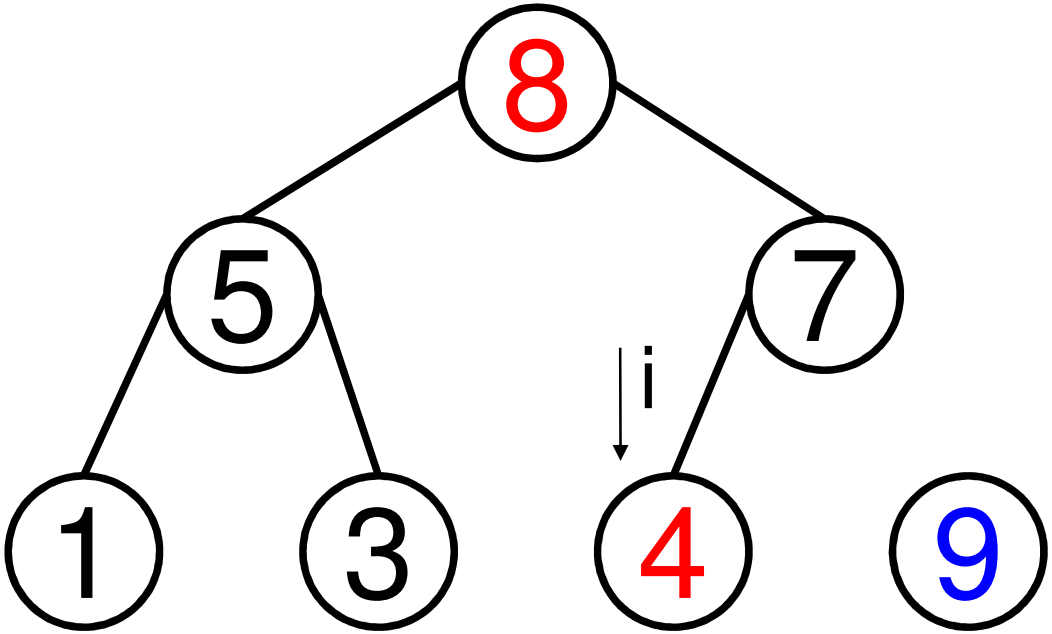
Heapsort

8	5	7	1	3	4	9



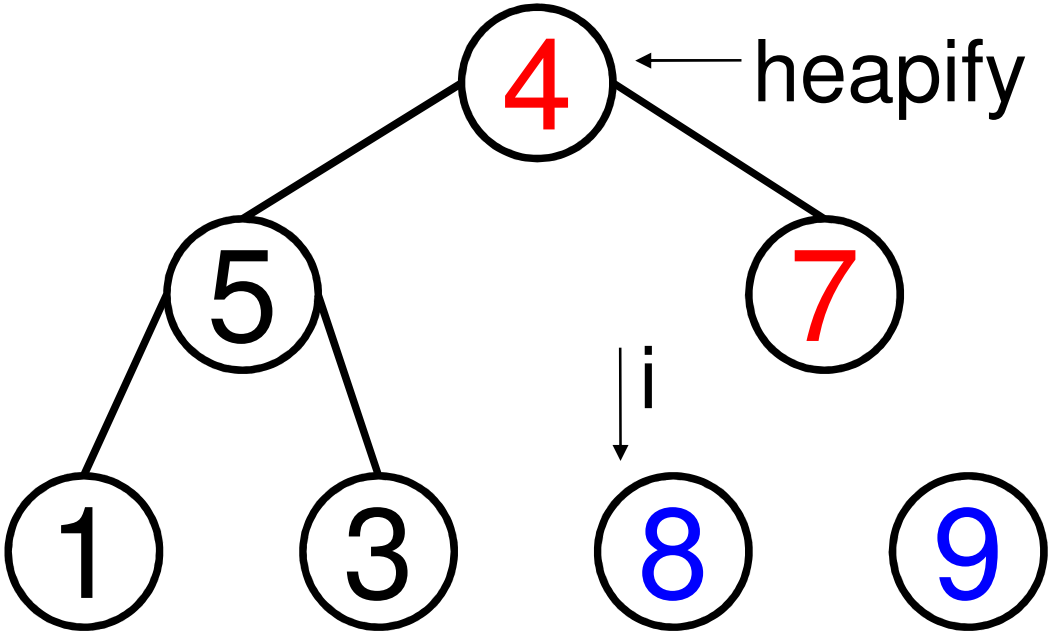
Heapsort

8	5	7	1	3	4	9



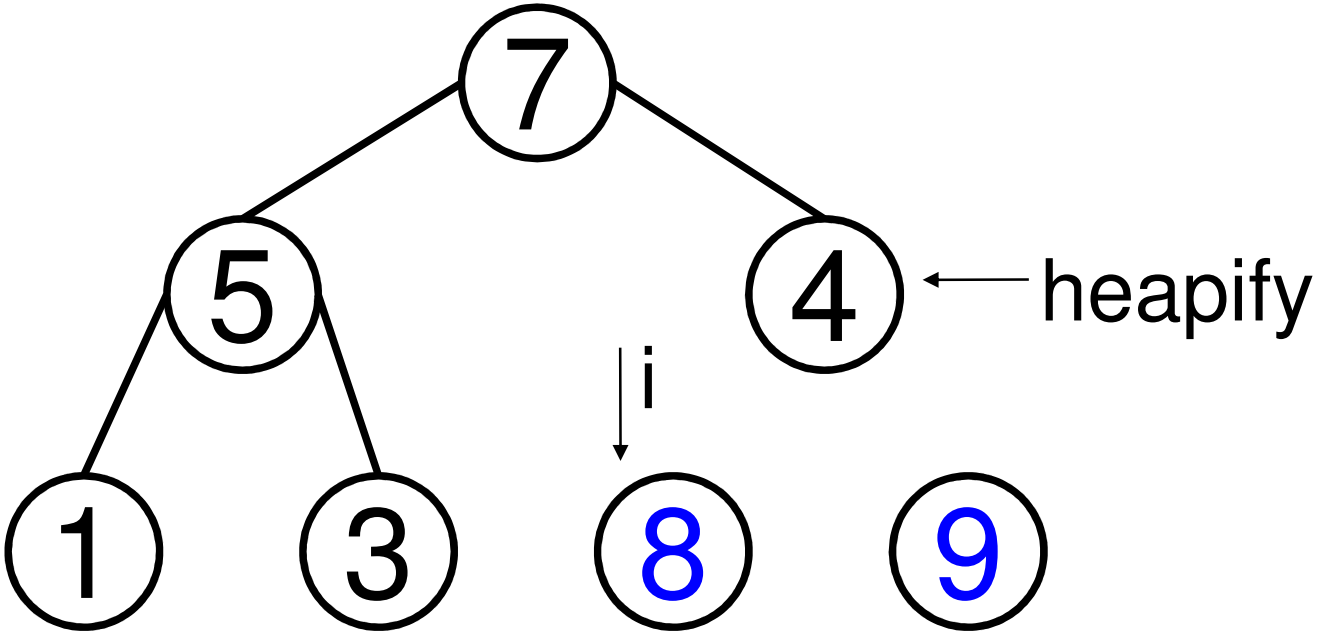
Heapsort

4	5	7	1	3	8	9



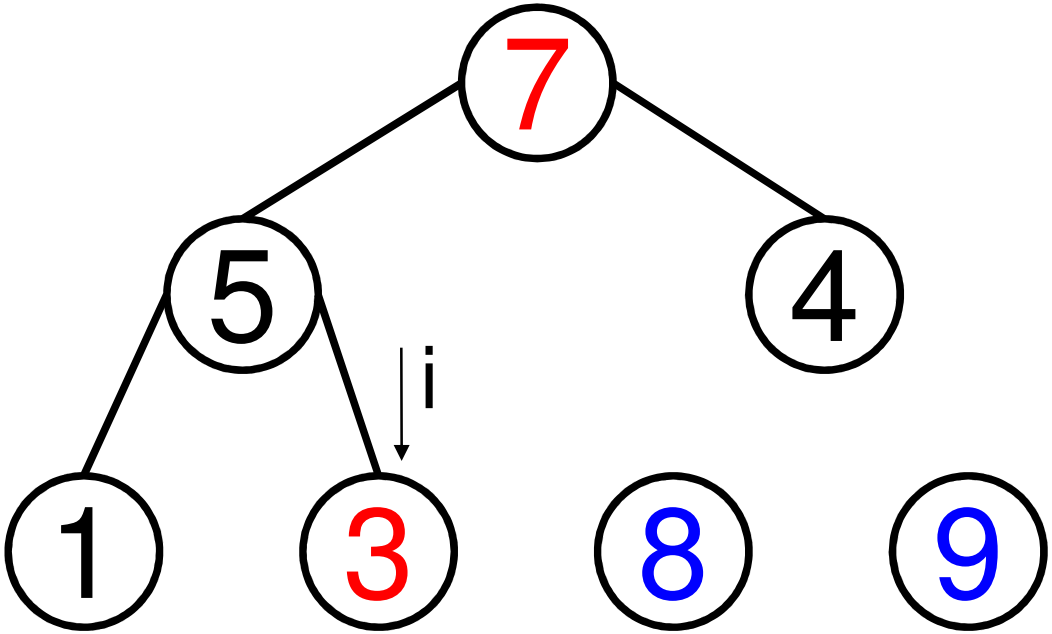
Heapsort

7	5	4	1	3	8	9

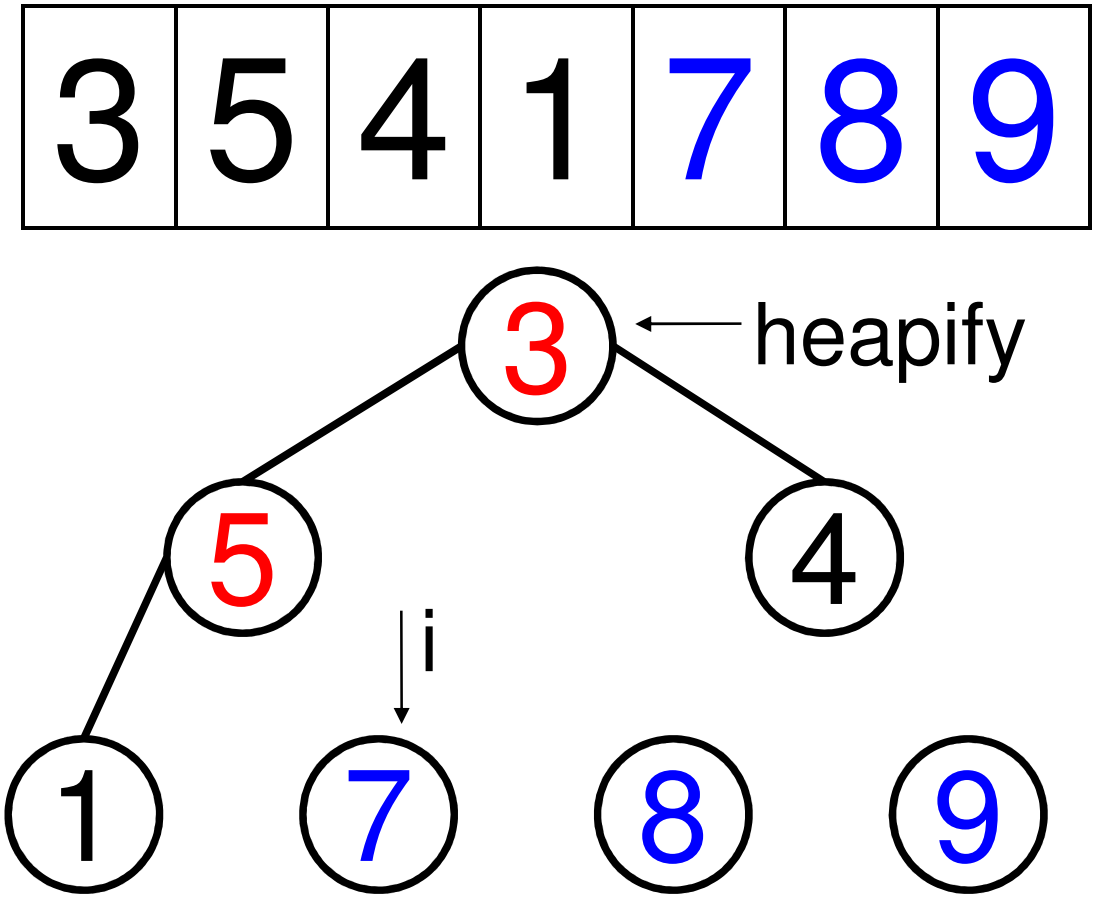


Heapsort

7	5	4	1	3	8	9

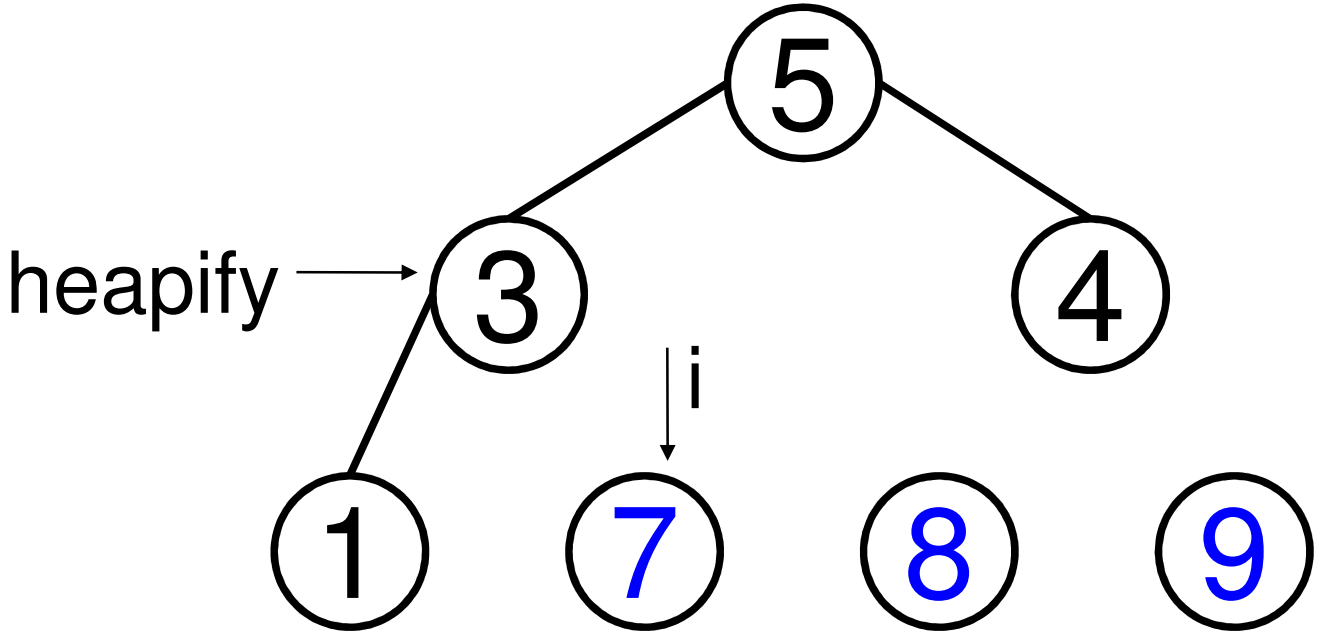


Heapsort



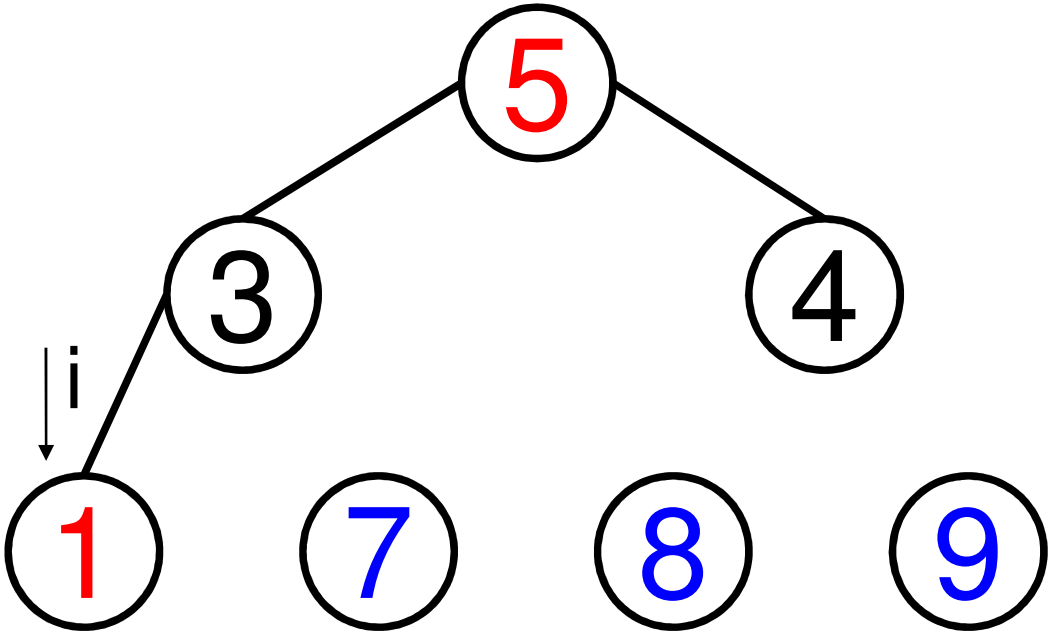
Heapsort

5	3	4	1	7	8	9

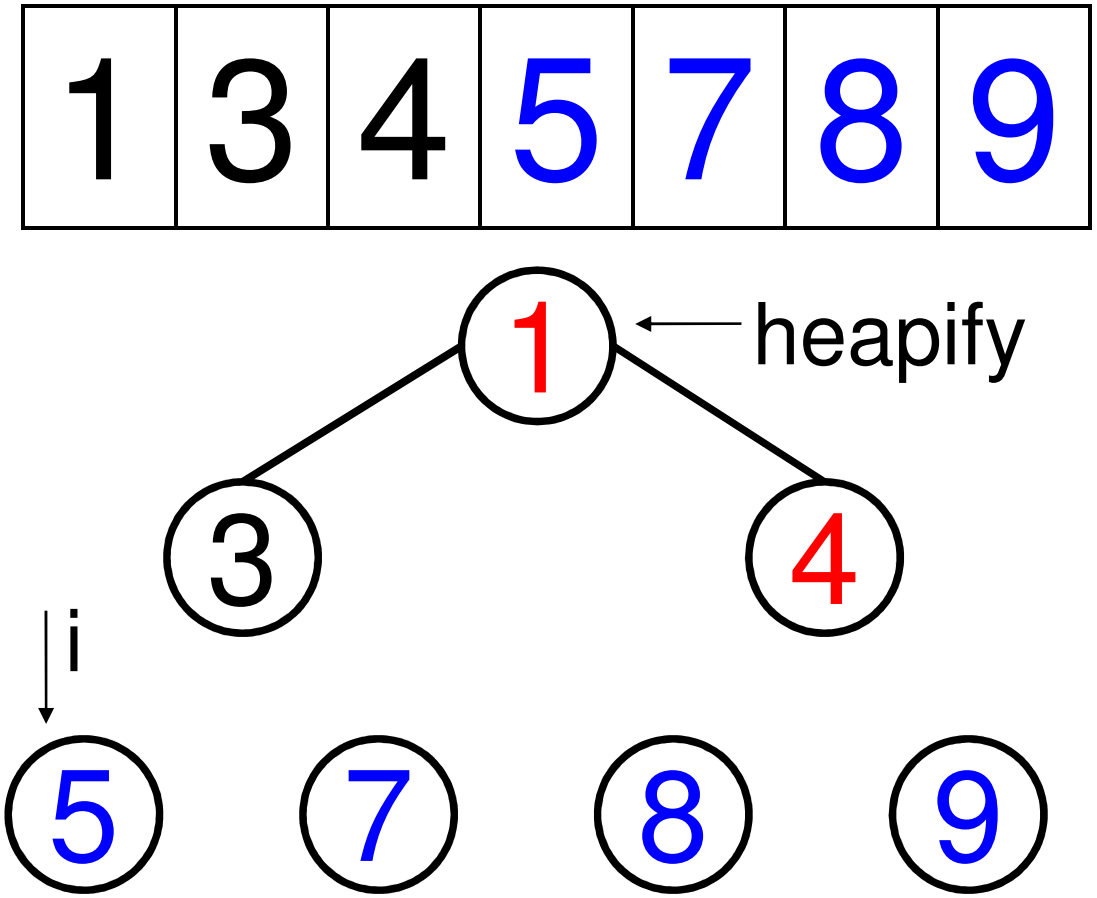


Heapsort

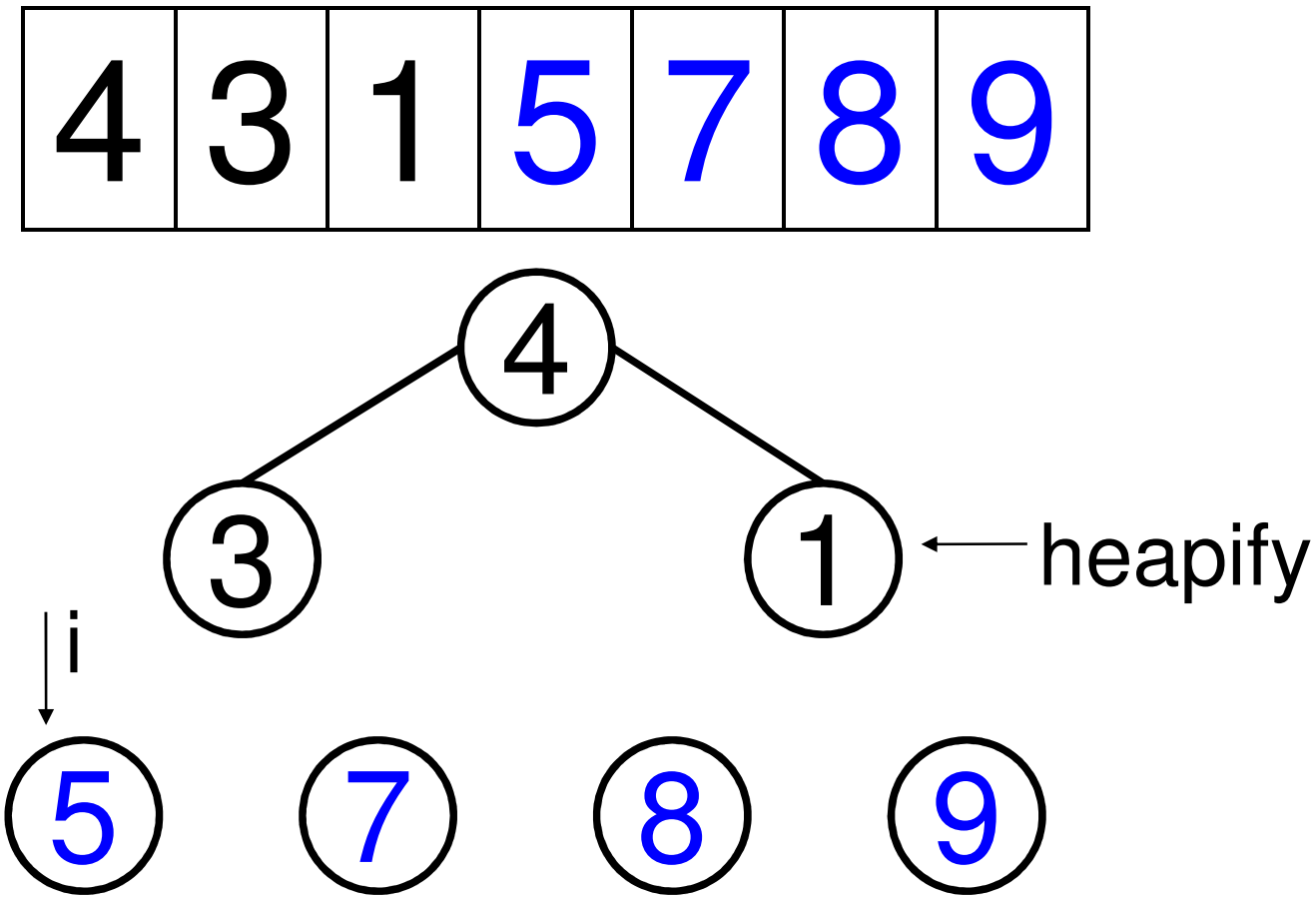
5	3	4	1	7	8	9



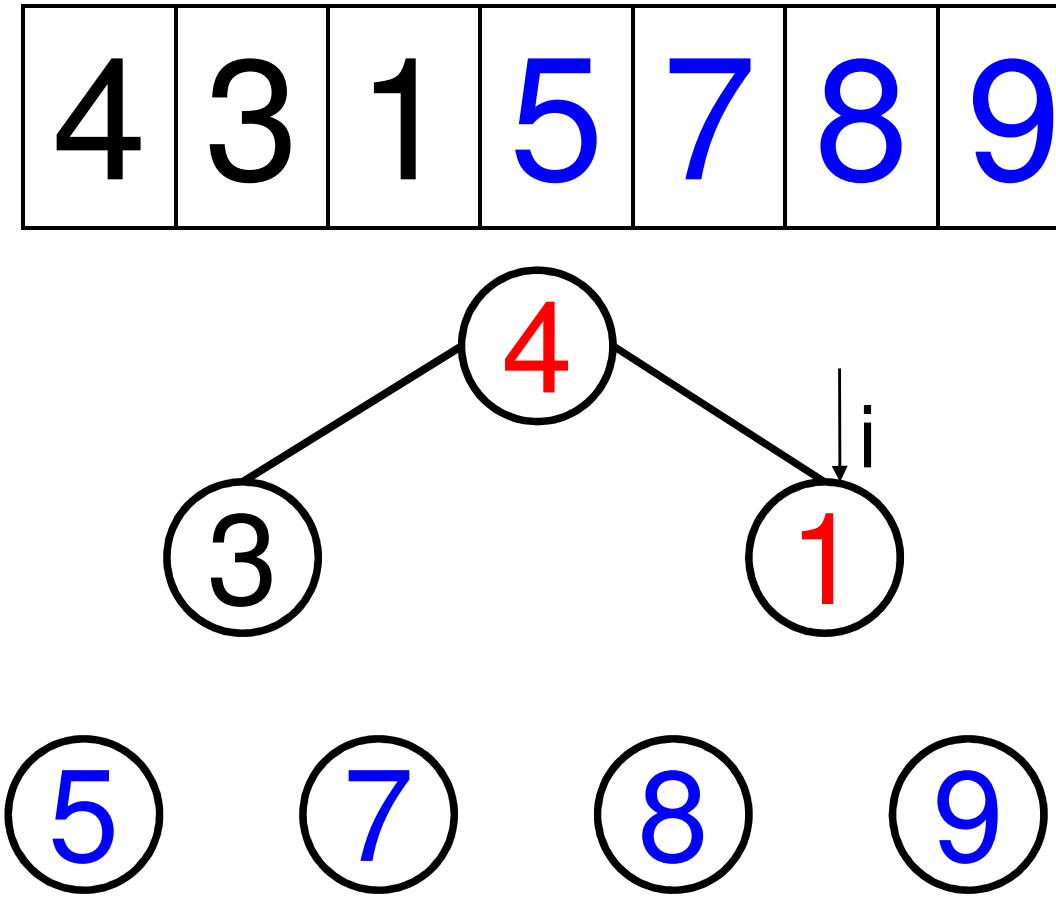
Heapsort



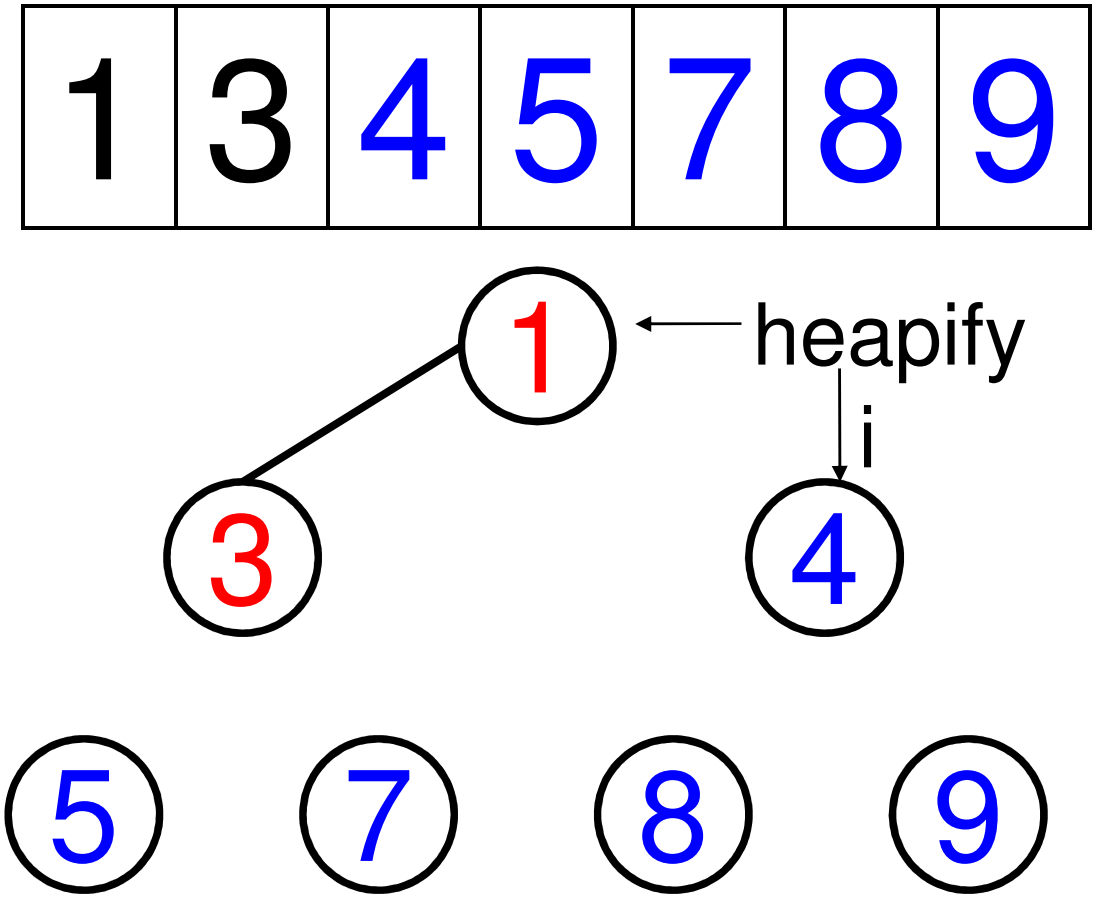
Heapsort



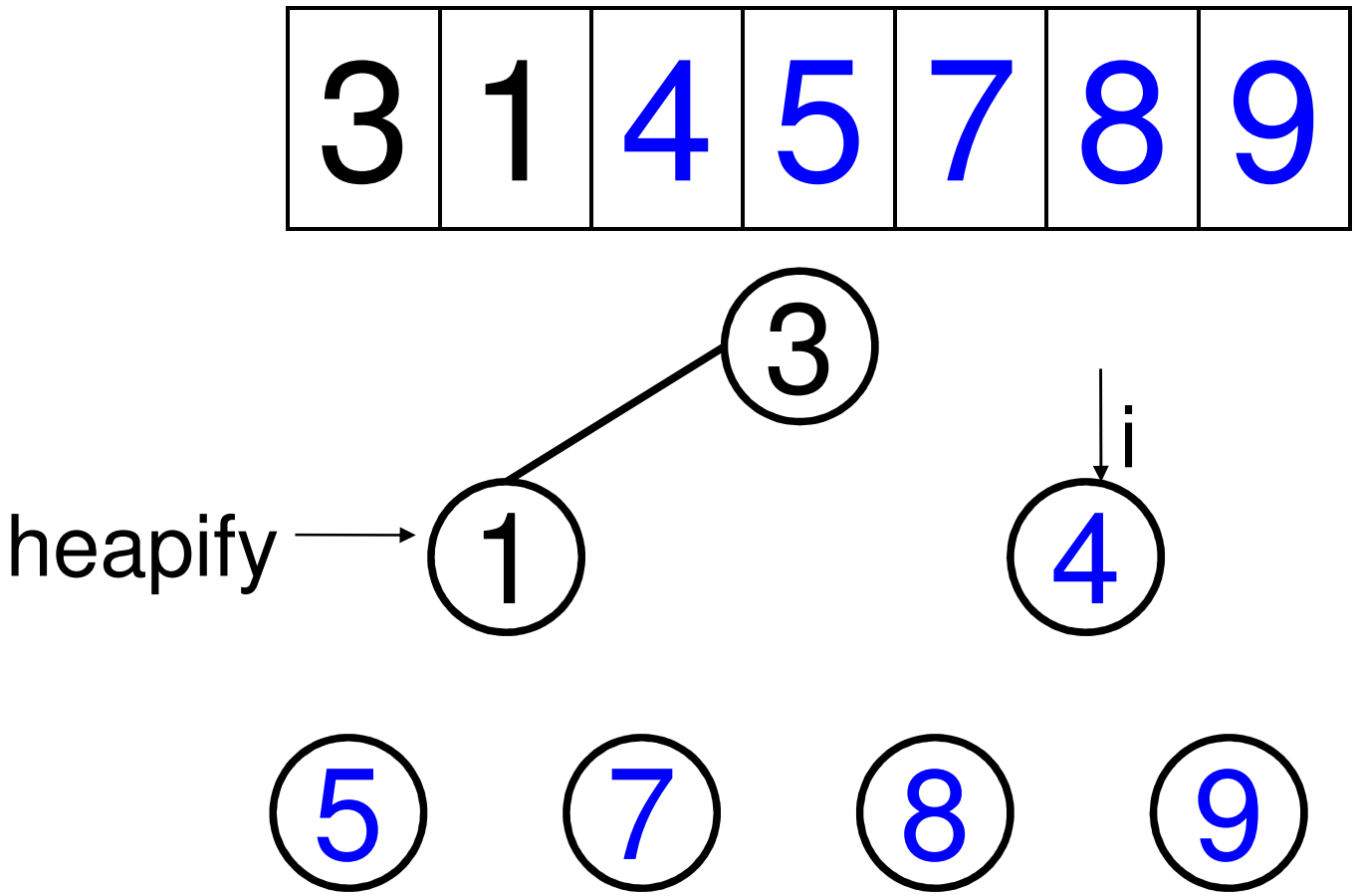
Heapsort



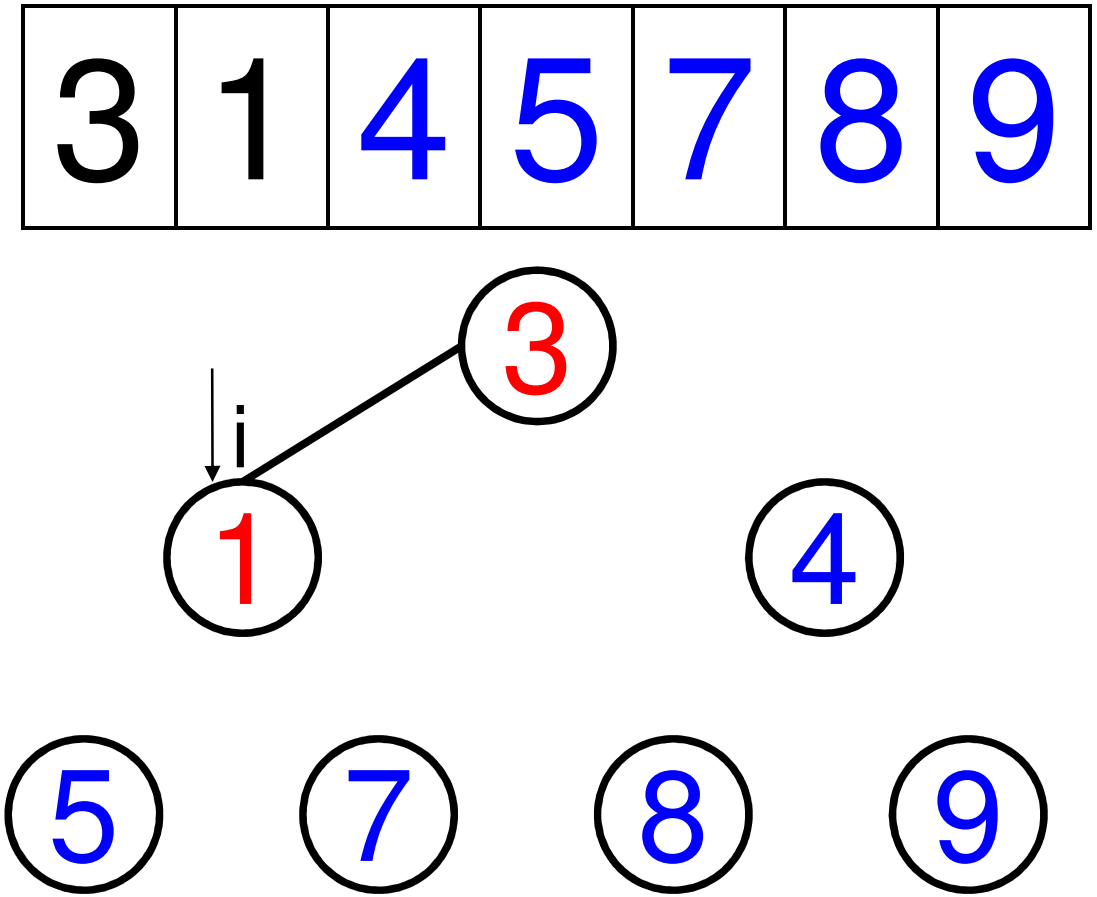
Heapsort



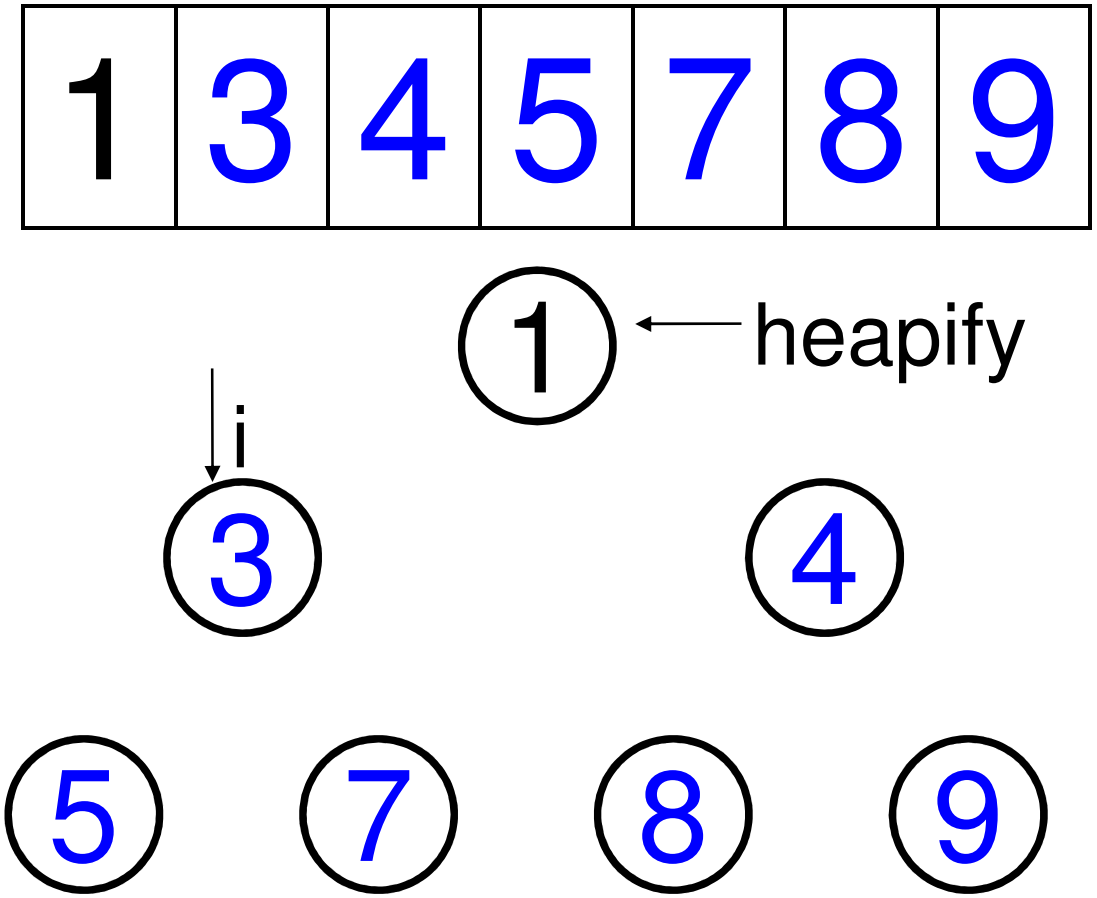
Heapsort



Heapsort

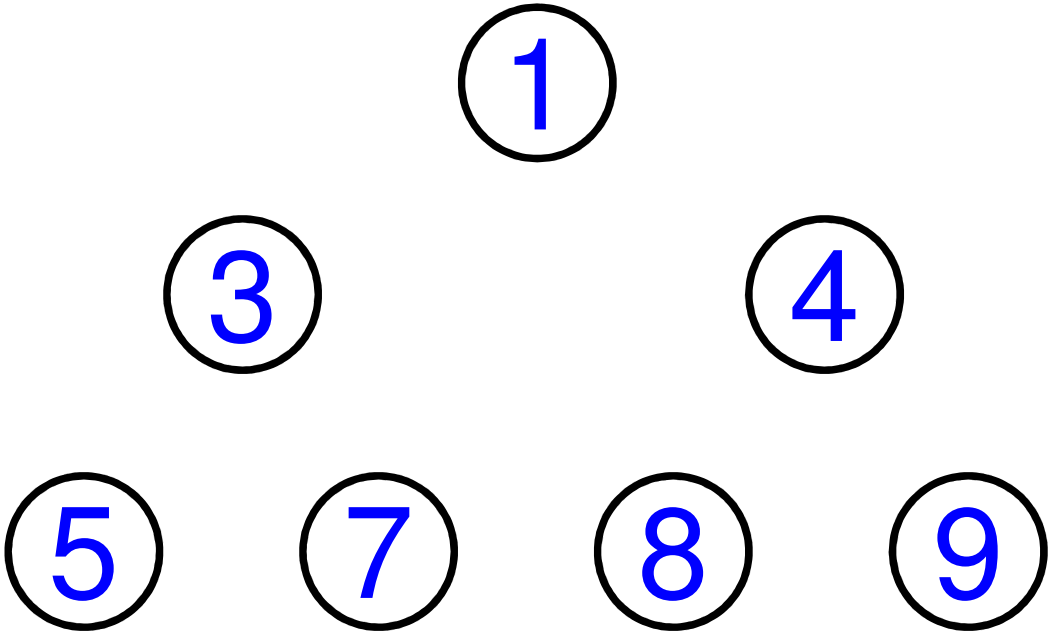


Heapsort

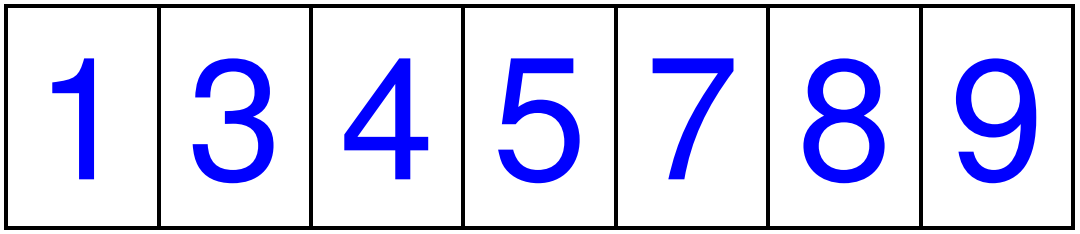


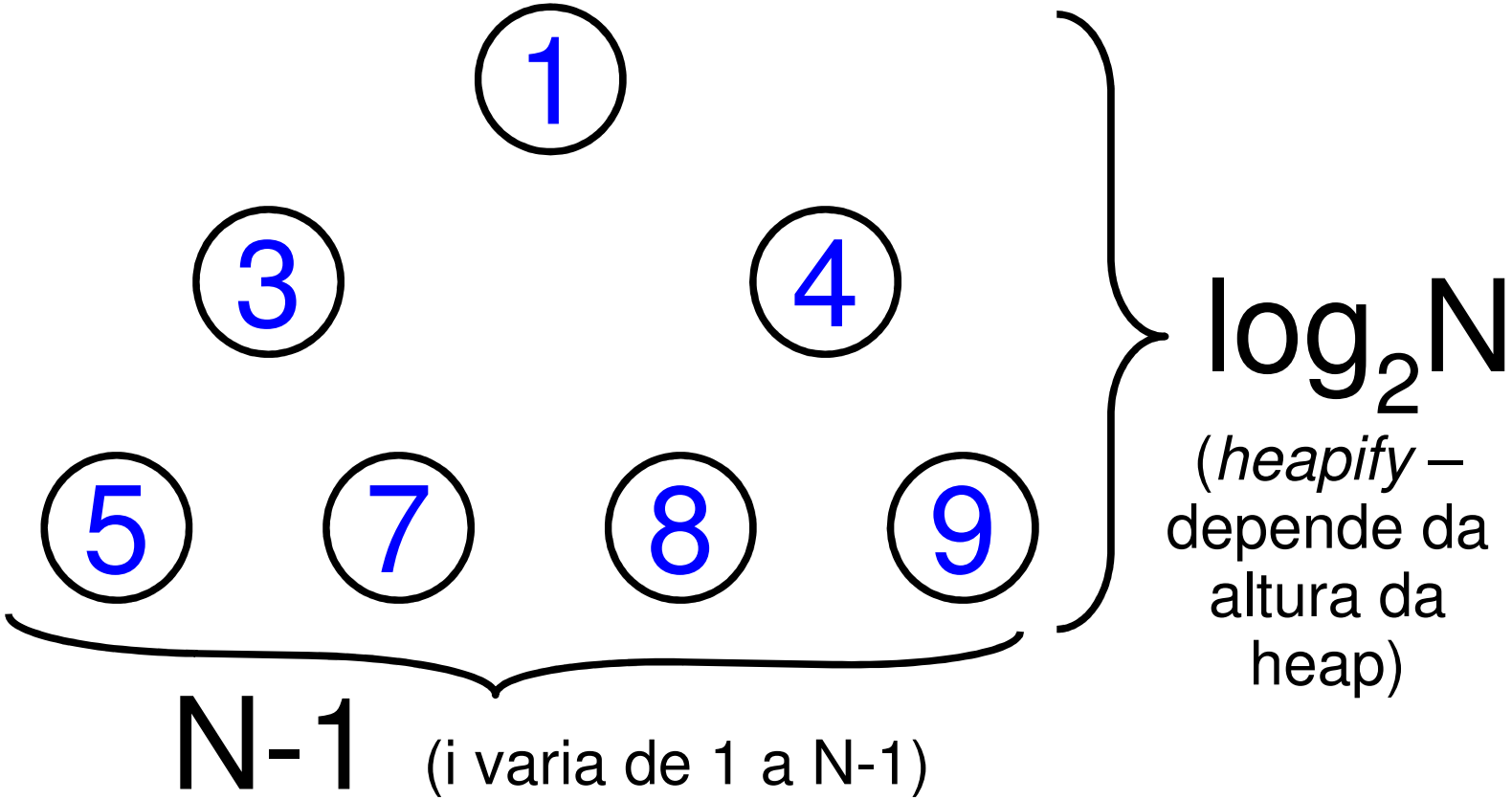
Heapsort

1	3	4	5	7	8	9

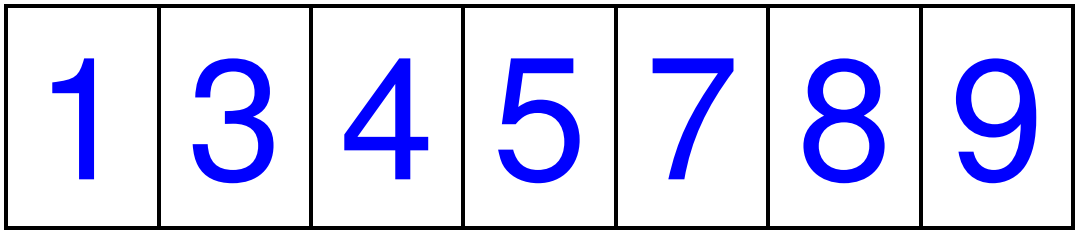


Operações do Heapsort

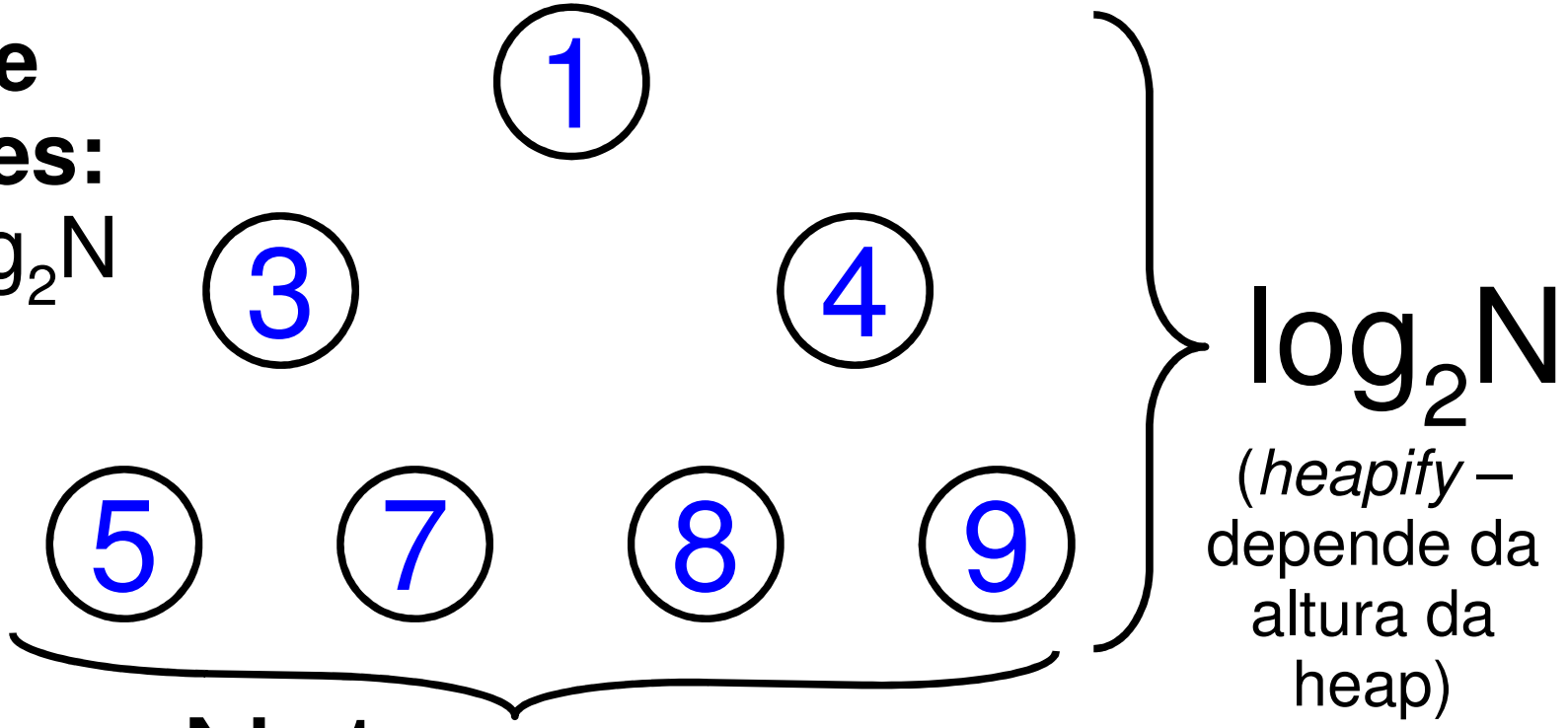




Operações do Heapsort



Total de operações:
$(N-1) * \log_2 N$



Quicksort

- Ordena o vetor particionando recursivamente
- A cada iteração, localiza a posição final de um elemento aleatório (**pivô**) e subdivide o vetor em duas partes para prosseguir a ordenação

Partição

- Localiza a posição final do **pivô** (um elemento aleatório do vetor) e divide o vetor em duas partes, garantindo:
  - Todos os elementos à esquerda do pivô são **menores ou iguais** a ele
  - Todos os elementos à direita do pivô são **maiores** que ele

Partição

```
function partition(V, left, right)
      pivot = V[left]
      i = left-1
      j = right+1
      while true do
         repeat j=j-1 until V[j]<=pivot
         repeat i=i+1 until V[i]>=pivot
         if i<j then
            V[i], V[j] = V[j], V[i]
         else
            return j
         end
      end
   end
```

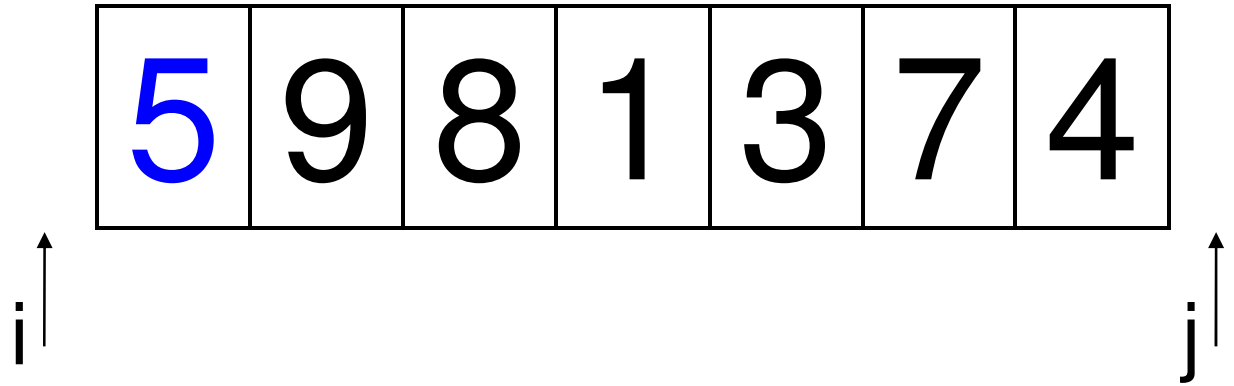
Quicksort

```
function quicksort(V, left, right)
    if left<right then
        middle = partition(V, left, right)
        quicksort(V, left, middle)
        quicksort(V, middle+1, right)
    end
end
```

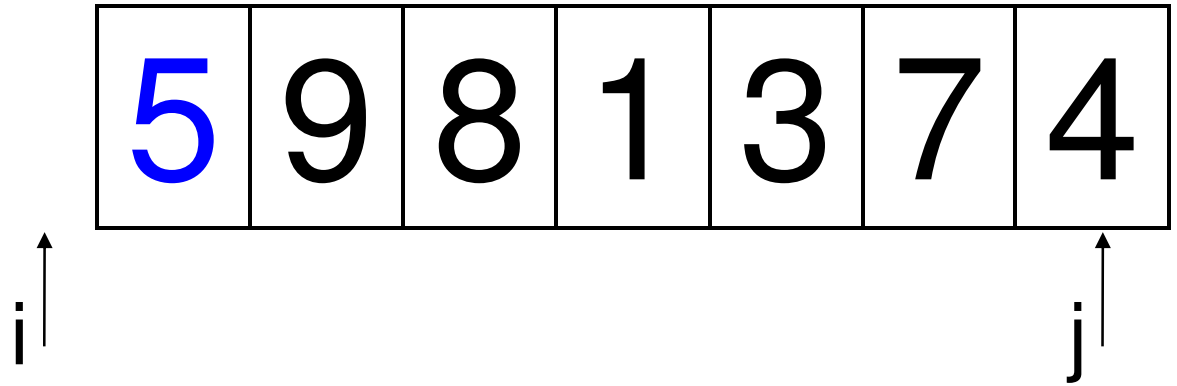
Quicksort - *quicksort*(1,7)

5	9	8	1	3	7	4
1	2	3	4	5	6	7

Quicksort - *partition*(1,7)

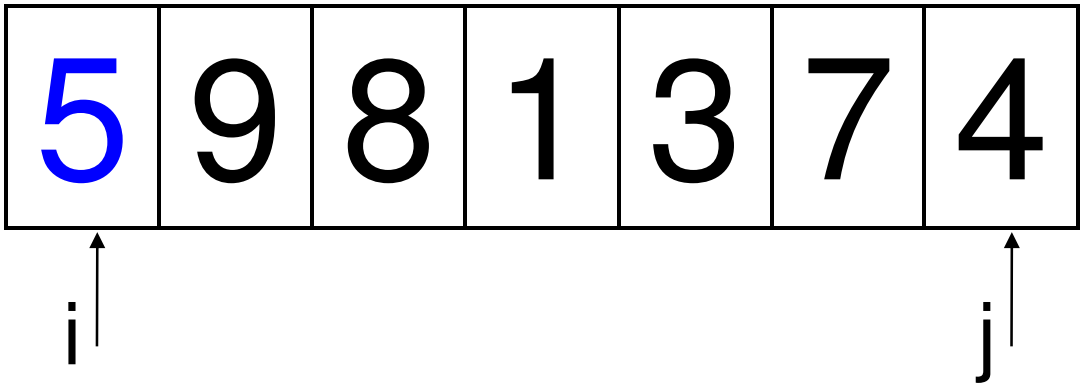


Quicksort - *partition*(1,7)



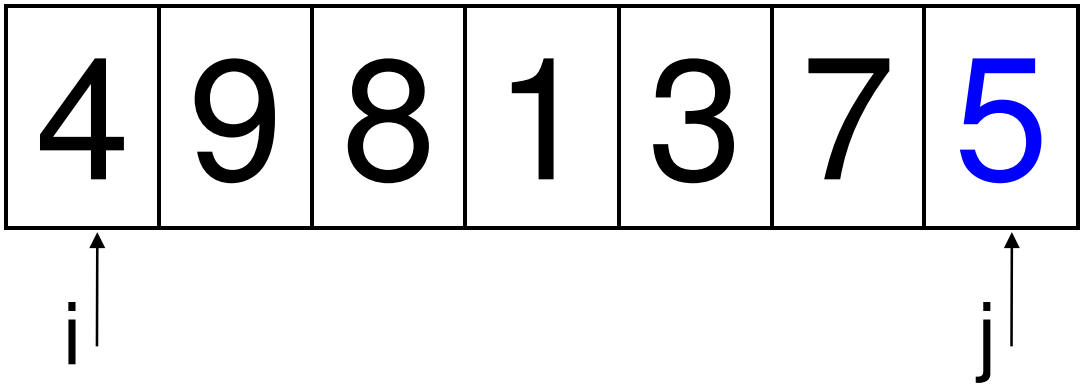
until V[j]<=5 ?

Quicksort - *partition*(1,7)



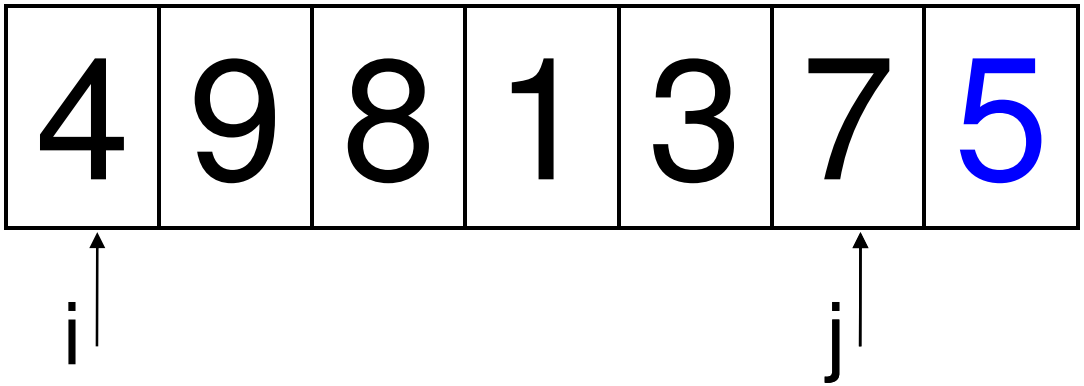
until V[i]>=5 ?

Quicksort - *partition*(1,7)



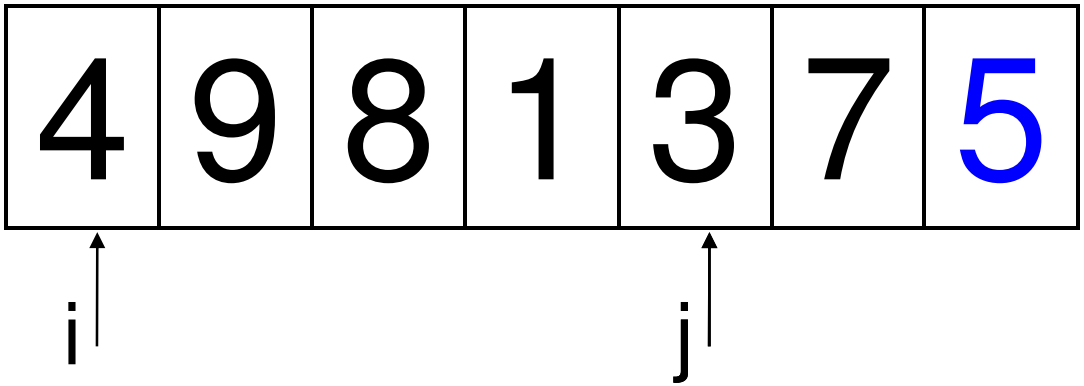
V[i], V[j] = V[j], V[i]

Quicksort - *partition*(1,7)



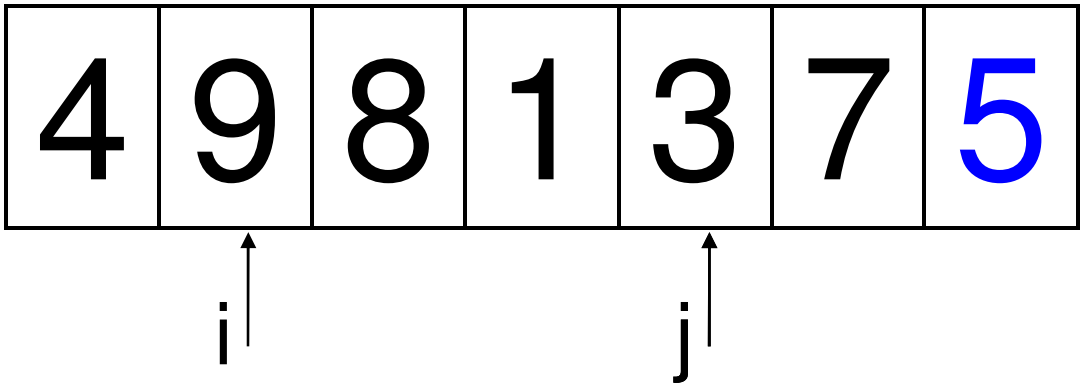
until V[j]<=5 ?

Quicksort - *partition*(1,7)



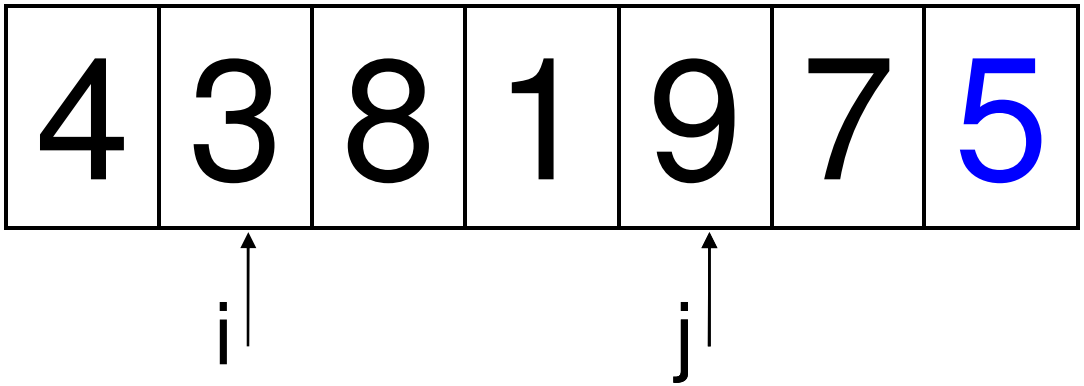
until V[j]<=5 ?

Quicksort - *partition*(1,7)



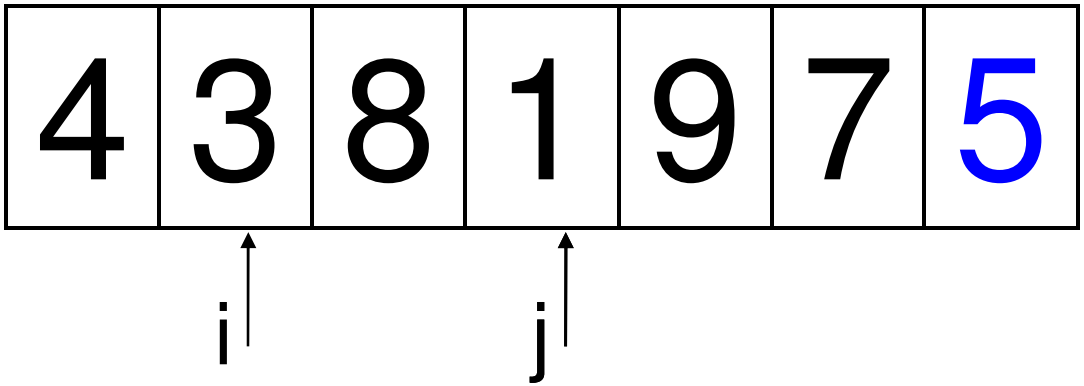
until V[i]>=5 ?

Quicksort - *partition*(1,7)



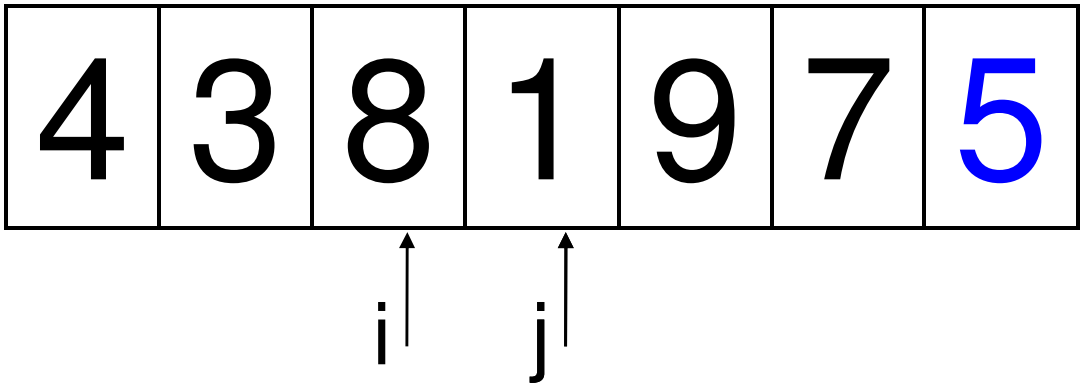
V[i], V[j] = V[j], V[i]

Quicksort - *partition*(1,7)



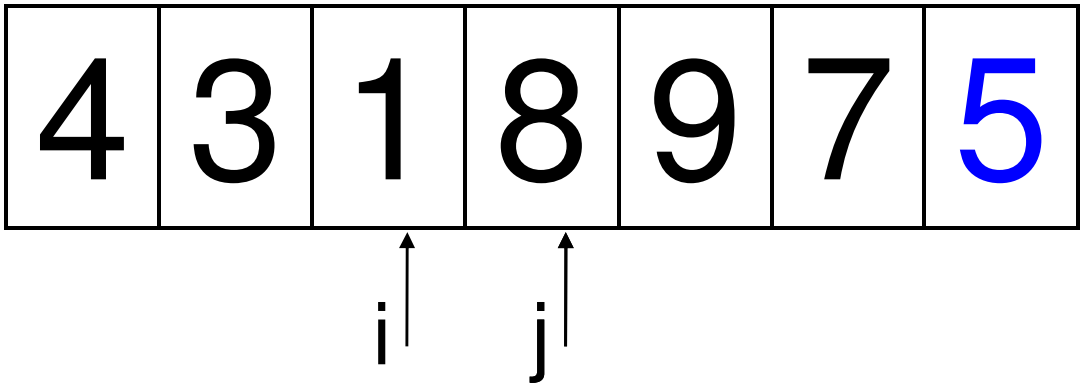
until V[j]<=5 ?

Quicksort - *partition*(1,7)



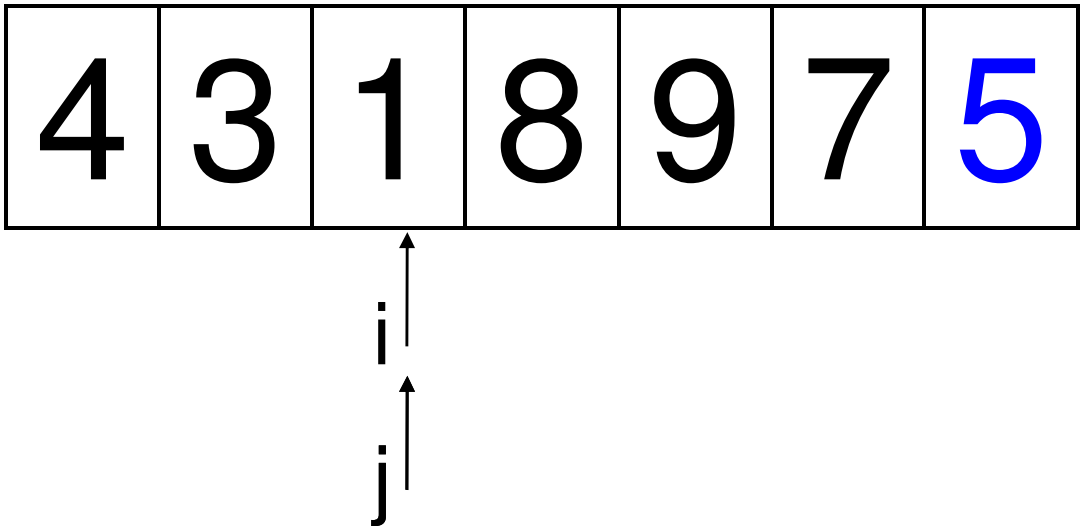
until V[i]>=5 ?

Quicksort - *partition*(1,7)



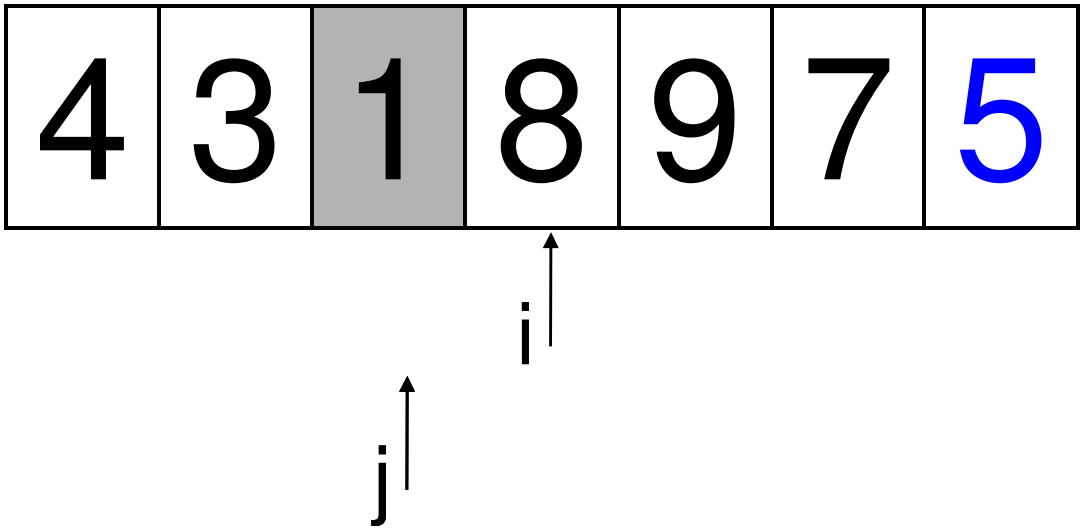
V[i], V[j] = V[j], V[i]

Quicksort - *partition*(1,7)



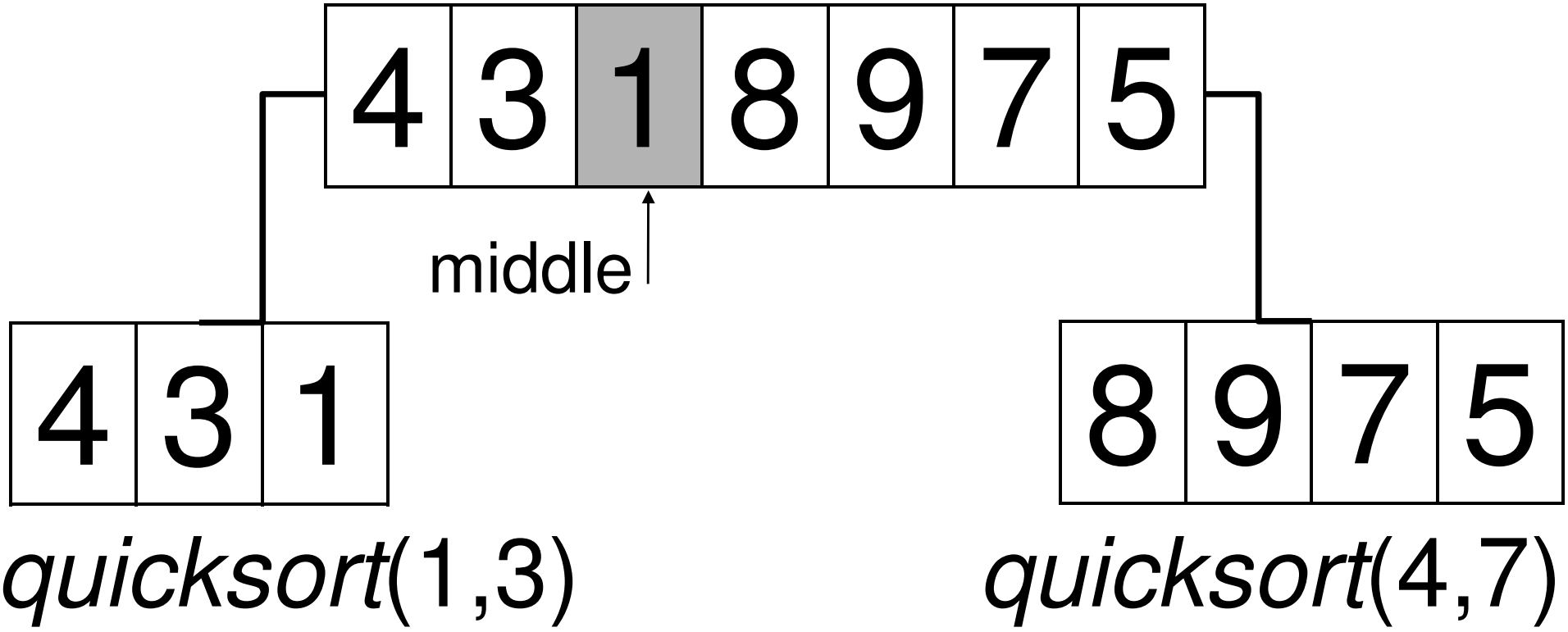
until V[j]<=5 ?

Quicksort - *partition*(1,7)

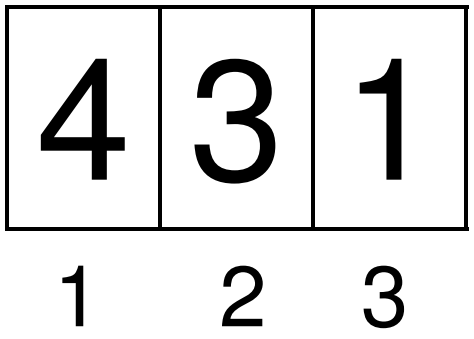


until V[i]>=5 ?

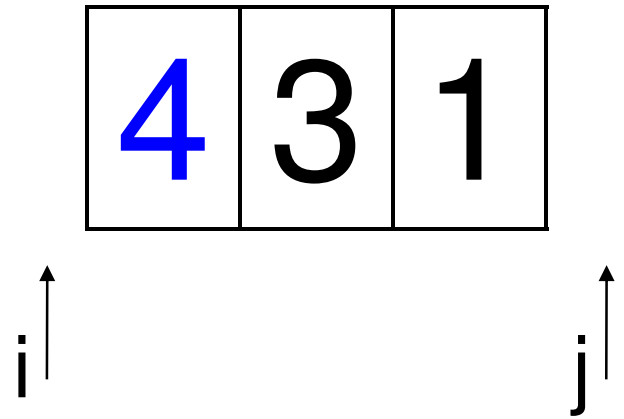
Quicksort - *quicksort*(1,7)



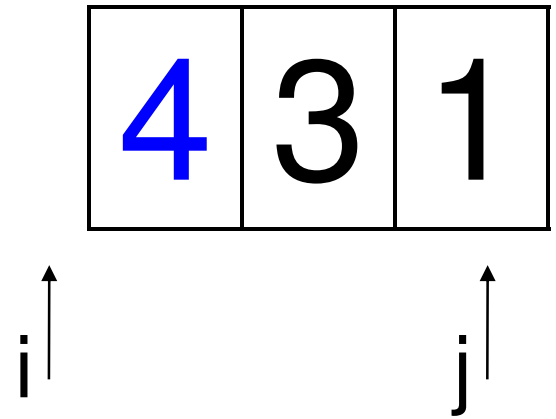
Quicksort - *quicksort*(1,3)



Quicksort - *partition*(1,3)

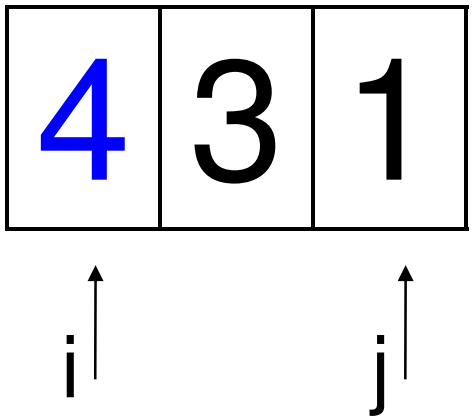


Quicksort - *partition*(1,3)



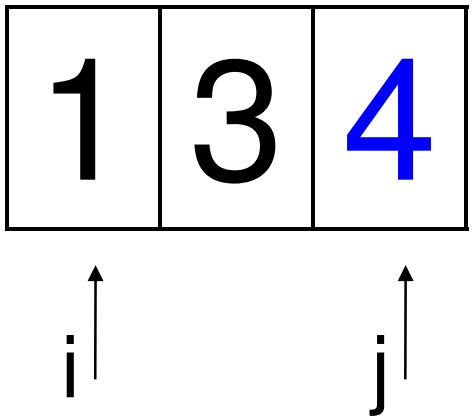
until V[j]<=4 ?

Quicksort - *partition*(1,3)



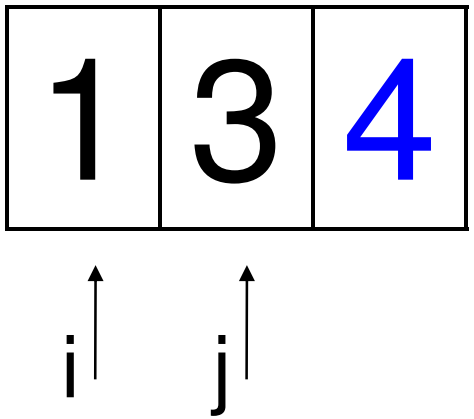
until V[i]>=4 ?

Quicksort - *partition*(1,3)



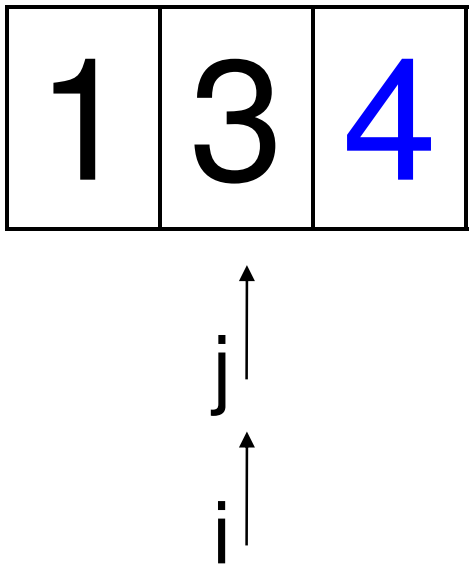
V[i], V[j] = V[j], V[i]

Quicksort - *partition*(1,3)



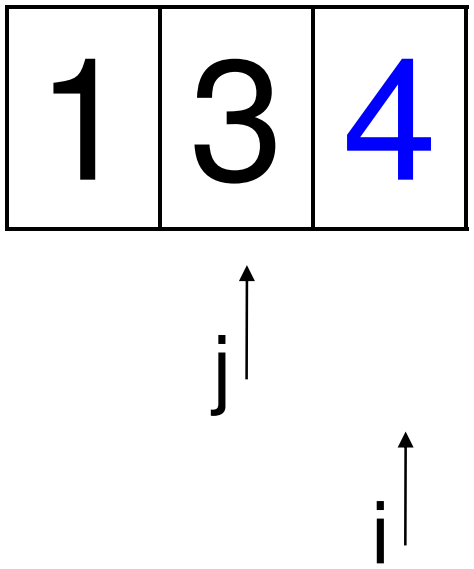
until V[j]<=4 ?

Quicksort - *partition*(1,3)



until V[i]>=4 ?

Quicksort - *partition*(1,3)



until V[i]>=4 ?

Quicksort - *partition*(1,3)